

## Electricity and Magnetism I (PHY 321)

### Lorentz Force Law problems

**Problem 1** A particle with mass  $m$  and charge  $q$  is in an electric field  $\mathbf{E}$  and a magnetic field  $\mathbf{B}$ . Use Newton's second law to write a vector differential equation for the velocity  $\mathbf{v}(t)$  of the particle. Write it as

$$\frac{d\mathbf{v}}{dt} =$$

Then take Cartesian components and write expressions for

$$\frac{dv_x}{dt} = \tag{1}$$

$$\frac{dv_y}{dt} = \tag{2}$$

$$\frac{dv_z}{dt} = \tag{3}$$

in terms of the eleven quantities  $m, q, v_x, v_y, v_z, E_x, E_y, E_z, B_x, B_y, B_z$ . Finally, say for each of the eleven quantities whether it is a constant, a function of time ( $t$ ), or a function of space-time ( $x, y, z$ , and  $t$ ).

**Problem 2** Consider a classical hydrogen atom consisting of a proton fixed at the origin and an electron free to move. Let  $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$  denote the position of the electron.

1. Write an expression for the force  $\mathbf{F}$  that the electron feels by being near the proton.
2. Write down Cartesian components of equations of motion for the electron in the form

$$\frac{d^2x}{dt^2} = \tag{4}$$

$$\frac{d^2y}{dt^2} = \tag{5}$$

$$\frac{d^2z}{dt^2} = \tag{6}$$

in terms of  $x, y, z$  and whatever else you need (but force, electric field, and electric potential should not appear explicitly).

**Problem 3** A particle with charge  $q$  and mass  $m$  passes through the origin of a system of coordinates at time  $t = 0$  with velocity  $\mathbf{v} = v_0 \hat{\mathbf{k}}$ . A uniform electric field  $\mathbf{E}(\mathbf{r}) = E_0 \hat{\mathbf{k}}$  and a uniform magnetic field  $\mathbf{B}(\mathbf{r}) = B_0 \hat{\mathbf{k}}$  are present in the vicinity of the of the charged particle. (This electric field and magnetic field are produced by *other* charges and currents.) Give expressions for the position  $\mathbf{r}(t)$  and the velocity  $\mathbf{v}(t)$  of the particle as functions of time.

**Problem 4** Consider an electric field

$$\mathbf{E}(\mathbf{r}) = \begin{cases} E_0 \hat{\mathbf{k}} & , \quad x < a \\ 0 & , \quad x \geq a \end{cases}$$

where  $a > 0$ . A particle with charge  $q$ , mass  $m$ , initial position  $\mathbf{r}(0) = 0$ , and initial velocity  $\mathbf{v}(0) = v_0 \hat{\mathbf{i}}$  travels through the electric field. ( $v_0$  is a constant with units of velocity.) Find the position  $\mathbf{r}(t)$  and velocity  $\mathbf{v}(t)$  of the particle as a function of time. There is no magnetic field present.

**Problem 5** Consider a uniform magnetic field  $\mathbf{B} = B_0 \hat{\mathbf{k}}$ , and no electric field. A particle with charge  $q$  and mass  $m$  begins in this magnetic field with a position  $\mathbf{r}(0) = 0$  and a velocity  $\mathbf{v}(0) = v_0 \hat{\mathbf{i}}$ . Find the position  $\mathbf{r}(t)$  and the velocity  $\mathbf{v}(t)$  of the particle at a later time  $t$ .

**Problem 6** Consider a uniform magnetic field  $\mathbf{B} = B_0 \hat{\mathbf{x}}$  and a uniform electric field  $\mathbf{E} = E_0 \hat{\mathbf{z}}$ . A particle with charge  $q$  and mass  $m$  begins in this electromagnetic field with a position  $\mathbf{r}(0) = 0$  and a velocity  $\mathbf{v}(0) = v_0 \hat{\mathbf{y}}$ .

- (a) Find the velocity  $\mathbf{v}(t)$  of the particle at a later time  $t$ .
- (b) Find  $v_x(t)$ ,  $v_y(t)$ , and  $v_z(t)$  for the particle in Example 5.2 of Griffiths (4th ed). If  $v_0 = 0$ , the result of part (a) should agree with that of Example 5.2. Confirm that this is true.
- (c) There is a special value of  $v_0$  which will cause the particle to travel in a straight line at constant velocity. Find this value of  $v_0$ .

**Problem 7** A particle with mass  $m$  and charge  $q$  moves in a region of space in which there is a uniform electric field  $\mathbf{E} = E_0\hat{\mathbf{z}}$  and a uniform magnetic field  $\mathbf{B} = B_0\hat{\mathbf{z}}$ . At time  $t = 0$ , the particle is passing through the origin with velocity  $\mathbf{v}(0) = v_0\hat{\mathbf{z}}$ .

- (a) Find the velocity  $\mathbf{v}(t)$  of the particle as a function of time.
- (b) Find the position  $\mathbf{r}(t)$  of the particle as a function of time.

**Problem 8** A particle with mass  $m$  and charge  $q$  moves in a region of space in which there is a uniform magnetic field  $\mathbf{B} = B_0\hat{\mathbf{z}}$ . No electric field is present. At time  $t = 0$ , the particle is passing through the origin with velocity  $\mathbf{v}(0) = v_0\hat{\mathbf{x}}$ .

- (a) Find the velocity  $\mathbf{v}(t)$  of the particle as a function of time.
- (b) Find the position  $\mathbf{r}(t)$  of the particle as a function of time.