The Maxwell Equations

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The Maxwell Equations (SI Units)

$$\vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_0} \rho \qquad \text{Gauss's Law}$$

$$\vec{\nabla} \times \vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{B}}}{\partial t} = 0 \qquad \text{Faraday's Law}$$

$$\vec{\nabla} \cdot \vec{\mathbf{B}} = 0 \qquad \text{no magnetic monopoles}$$

$$\vec{\nabla} \times \vec{\mathbf{B}} - \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} = \mu_0 \vec{\mathbf{J}} \qquad \text{Ampere-Maxwell Law}$$

Integral form of the Maxwell equations (SI Units)

$$\int_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{a}} = \frac{Q}{\epsilon_0}$$ Gauss's Law

$$\int_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d\Phi_B}{dt}$$ Faraday's Law

$$\int_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}} = 0$$ no magnetic monopoles

$$\int_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$ Ampere-Maxwell Law

The Maxwell equations in matter

$\vec{\nabla} \cdot \vec{\mathbf{D}} = \rho_f$	Gauss's Law
$\vec{\nabla} \times \vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{B}}}{\partial t} = 0$	Faraday's Law
$\vec{\nabla} \cdot \vec{\mathbf{B}} = 0$	no magnetic monopoles
$\vec{\nabla} \times \vec{\mathbf{H}} - \frac{\partial \vec{\mathbf{D}}}{\partial t} = \vec{\mathbf{J}}_f$	Ampere-Maxwell Law

Integral form of the Maxwell equations in matter

$\int_S \vec{\mathbf{D}} \cdot d\vec{\mathbf{a}} = Q_f$	Gauss's Law
$\int_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d\Phi_B}{dt}$	Faraday's Law
$\int_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}} = 0$	no magnetic monopoles
$\int_C \vec{\mathbf{H}} \cdot d\vec{\mathbf{l}} = I_f + \frac{d\Phi_D}{dt}$	Ampere-Maxwell Law