Physics 321

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Electric field from a line charge

$$\vec{E}(x,y,z) = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(x',y',z')\hat{\boldsymbol{\mathscr{r}}}\,dl'}{\mathscr{r}^2}$$

- for Coulomb's Law problems 12 and 20
- dl' is usually one of the following:
  - dx
  - dy
  - dz
  - ds
  - $s\,d\phi$
  - dr
  - $r\,d\theta$
  - $r\sin\theta\,d\phi$

Electric field produced by a line charge

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(\vec{r}\,')\hat{\mathscr{r}}\,dl'}{\mathscr{r}^2}$$
$$= \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(\vec{r}\,')(\vec{r}-\vec{r}\,')\,dl'}{|\vec{r}-\vec{r}\,'|^3}$$

- $\vec{r}$ = field point (place where we are finding the electric field)
- $\vec{r}\,'$ = source point (location of charge that we are integrating over)

Electric field from a line charge - setting up the integral

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(\vec{r}\,')\hat{\boldsymbol{\mathscr{r}}}\,dl'}{\mathscr{r}^2}$$
$$= \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(\vec{r}\,')(\vec{r}-\vec{r}\,')\,dl'}{|\vec{r}-\vec{r}\,'|^3}$$

Item	from
$\vec{r}$	where we want to know the electric field
$\vec{r}\,'$	curve describing the location of the charge
dl'	curve describing the location of the charge
$\lambda(\vec{r}\,')$	given linear charge density

Curve 1

The path from $(0, 0, 0)$ to $(x_0, 0, 0)$ along the x axis, then to $(x_0, y_0, 0)$ along a straight line parallel to the y axis, then to (x_0, y_0, z_0) along a straight line parallel to the z axis.

$$\vec{r}' = ?$$
$$dl' = ?$$

Curve 2

The straight-line path from $(0, 0, 0)$ to (a, a, a) where a is a constant with dimensions of length.

$$\vec{\mathrm{r}}' = ?$$

$$dl' = ?$$

Curve 3

The path around a square from $(0,0,0)$ to $(a,0,0)$ to $(a,a,0)$ to $(0,a,0)$ to $(0,0,0)$ where a is a constant with dimensions of length.

$$\vec{r}' = ?$$
$$dl' = ?$$

Curve 4

The circular path with radius R in the $z = 0$ plane starting at $\phi = 0$ and going to $\phi = 2\pi$.

$$\vec{r}' = ?$$

$$dl' = ?$$

Curve 5

The straight path in the $z = 0$ plane from the origin along $\phi = \pi/4$ until $s = R$.

$$\vec{\mathrm{r}}' = ?$$

$$dl' = ?$$

Curve 6

The path on the surface of a cylinder of radius R that goes (i) along a circular arc from a point at $(x, y, z) = (R, 0, h)$ to a point at $(x, y, z) = (0, R, h)$, and then (ii) along a straight-line path from $(x, y, z) = (0, R, h)$ to $(x, y, z) = (0, R, 0)$.

$$\vec{\mathrm{r}}' = ?$$

$$dl' = ?$$

Curve 7

The path on the surface of a sphere of radius R that goes (i) from the north pole at $(x, y, z) = (0, 0, R)$ to the equator at $(x, y, z) = (R, 0, 0)$, and then (ii) along the equator to the point $(x, y, z) = (R/\sqrt{2}, R/\sqrt{2}, 0)$.

$$\vec{r}' = ?$$
$$dl' = ?$$

Curve 8

The straight-line path from the origin to the point with spherical coordinates $(r, \theta, \phi) = (2, \pi/6, \pi/4)$.

$$\vec{r}' = ?$$

$$dl' = ?$$

Curve 9

The path from $(r, \theta, \phi) = (R, \pi/4, 0)$ to $(r, \theta, \phi) = (R, \pi/4, \pi/2)$ along which $r = R$ and $\theta = \pi/4$.

$$\vec{\mathrm{r}}' = ?$$

$$dl' = ?$$

Curve 10

The path from $(r, \theta, \phi) = (R, 0, 2\pi/3)$ to $(r, \theta, \phi) = (R, \pi/2, 2\pi/3)$ along which $r = R$ and $\phi = 2\pi/3$.

$$\vec{\mathrm{r}}' = ?$$

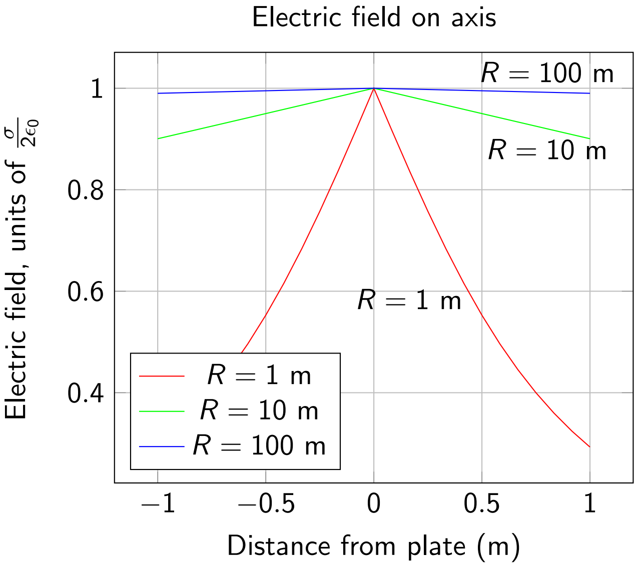
$$dl' = ?$$

Electric field from a surface charge

$$\vec{E}(x,y,z) = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(x',y',z')\,\hat{\boldsymbol{\mathscr{r}}}\,da'}{\mathscr{r}^2}$$

- for Coulomb's Law problem 13
- da' is usually one of the following:
  - $dx\,dy$
  - $dx\,dz$
  - $dy\,dz$
  - $s\,ds\,d\phi$
  - $ds\,dz$
  - $s\,d\phi\,dz$
  - $r\,dr\,d\theta$
  - $r\sin\theta\,dr\,d\phi$
  - $r^2\sin\theta\,d\theta\,d\phi$

Uniformly charged flat disk with radius R



Total charge

The path P is along the line charge. The linear charge density of the line charge is denoted λ, and the total charge is Q.

$$Q = \int_P \lambda(\vec{r}') \, dl' \tag{1}$$

The surface S is over the surface charge.

$$Q = \int_S \sigma(\vec{r}') \, da' \tag{2}$$

The volume V is over the volume charge.

$$Q = \int_V \rho(\vec{r}') \, dv' \tag{3}$$

Edge effects