Cartesian in Cylindrical

$$x = s\cos\phi$$
$$y = s\sin\phi$$
$$z = z$$

$$\hat{\mathbf{i}} = \cos\phi\hat{\mathbf{s}} - \sin\phi\hat{\boldsymbol{\phi}}$$
$$\hat{\mathbf{j}} = \sin\phi\hat{\mathbf{s}} + \cos\phi\hat{\boldsymbol{\phi}}$$
$$\hat{\mathbf{k}} = \hat{\mathbf{k}}$$

Cartesian in Spherical

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

$$\hat{\mathbf{i}} = \sin\theta\cos\phi\hat{\mathbf{r}} + \cos\theta\cos\phi\hat{\boldsymbol{\theta}} - \sin\phi\hat{\boldsymbol{\phi}}$$
$$\hat{\mathbf{j}} = \sin\theta\sin\phi\hat{\mathbf{r}} + \cos\theta\sin\phi\hat{\boldsymbol{\theta}} + \cos\phi\hat{\boldsymbol{\phi}}$$
$$\hat{\mathbf{k}} = \cos\theta\hat{\mathbf{r}} - \sin\theta\hat{\boldsymbol{\theta}}$$

Cylindrical in Spherical

$$s = r\sin\theta$$
$$\phi = \phi$$
$$z = r\cos\theta$$

Cartesian

$$d\mathbf{r} = dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}}$$

$$\nabla f = \hat{\mathbf{i}}\frac{\partial f}{\partial x} + \hat{\mathbf{j}}\frac{\partial f}{\partial y} + \hat{\mathbf{k}}\frac{\partial f}{\partial z}$$

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\nabla \times \mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\mathbf{i}} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\mathbf{j}} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{\mathbf{k}}$$

Cylindrical

$$d\mathbf{r} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{k}}$$

$$\nabla f = \hat{\mathbf{s}}\frac{\partial f}{\partial s} + \hat{\boldsymbol{\phi}}\frac{1}{s}\frac{\partial f}{\partial \phi} + \hat{\mathbf{k}}\frac{\partial f}{\partial z}$$

$$\nabla \cdot \mathbf{F} = \frac{1}{s}\frac{\partial (sF_s)}{\partial s} + \frac{1}{s}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$$

$$\nabla \times \mathbf{F} = \left(\frac{1}{s}\frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}\right)\hat{\mathbf{s}} + \left(\frac{\partial F_s}{\partial z} - \frac{\partial F_z}{\partial s}\right)\hat{\boldsymbol{\phi}} + \frac{1}{s}\left(\frac{\partial (sF_\phi)}{\partial s} - \frac{\partial F_s}{\partial \phi}\right)\hat{\mathbf{k}}$$

Spherical

$$d\mathbf{r} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$$

$$\nabla f = \hat{\mathbf{r}}\frac{\partial f}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial f}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}$$

$$\nabla \cdot \mathbf{F} = \frac{1}{r^2}\frac{\partial (r^2 F_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (\sin\theta F_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{F} = \frac{1}{r\sin\theta}\left(\frac{\partial (\sin\theta\,F_\phi)}{\partial \theta} - \frac{\partial F_\theta}{\partial \phi}\right)\hat{\mathbf{r}} + \left(\frac{1}{r\sin\theta}\frac{\partial F_r}{\partial \phi} - \frac{1}{r}\frac{\partial (rF_\phi)}{\partial r}\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial (rF_\theta)}{\partial r} - \frac{\partial F_r}{\partial \theta}\right)\hat{\boldsymbol{\phi}}$$