Work

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Work is force times distance

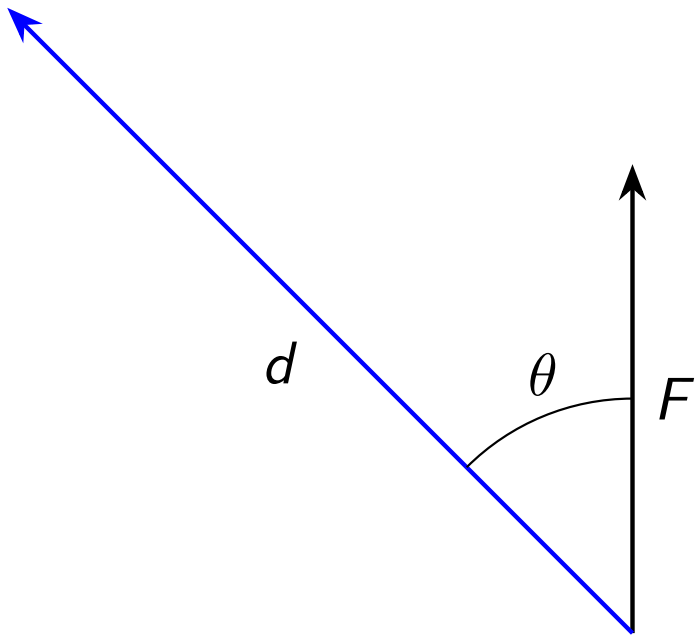
In introductory physics, we say that work is force times distance (or force times displacement).

$$W = Fd$$

Work

A force can do work on an object. Work, like energy, is measured in Joules (J). The work done by a force F on an object experiencing a displacement d is

$$W = Fd\cos\theta.$$



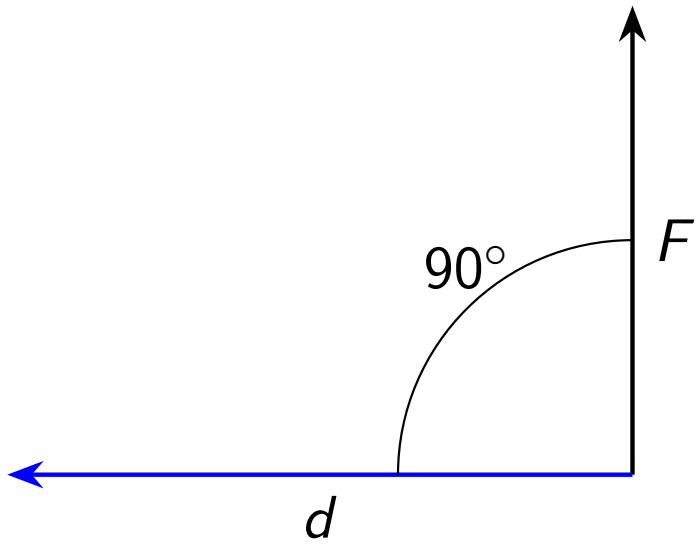
No Displacement, No Work

If an object doesn't move, forces may be acting on it, but none of them do any work.

$$W = Fd\cos\theta$$

If $d = 0$, then $W = 0$.

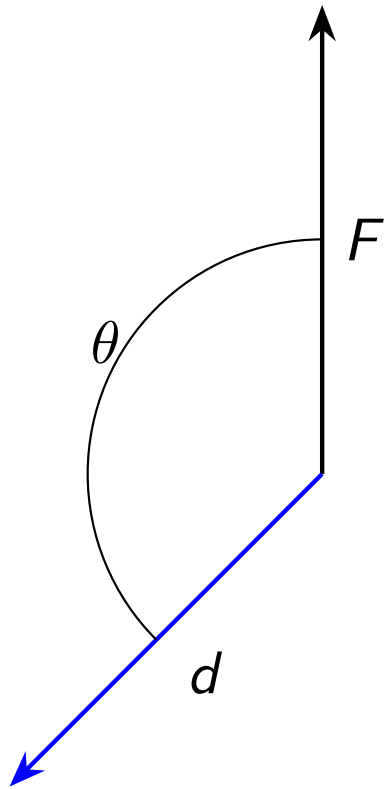
A force perpendicular to displacement does no work.



$$W = Fd\cos\theta$$

If $\theta = 90°$, then $W = 0$.

A force can do negative work.



$$W = Fd\cos\theta$$

If $\theta > 90°$, then $W < 0$.

The dot product is perfect for work.

$$W = Fd\cos\theta = \vec{F} \cdot \vec{d}$$

What if the force changes over the course of the displacement?

Instead of

$$W = \vec{F} \cdot \vec{d}$$

we have

$$W = \int_C \vec{F} \cdot d\vec{\ell}.$$

Finally we can tell you the truth: work is a dotted line integral.

Definition of work done by a force

If a force acts on an object as it moves along a curve C, the work done by the force is

$$W = \int_C \vec{F} \cdot d\vec{\ell}.$$

The force $\vec{F}$ can be conservative or non-conservative.