Lorentz Force Law

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Modern Electromagnetic Theory

- The Maxwell Equations

$$\vec{\nabla} \times \vec{B} - \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J} \qquad \vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$$

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$

- The Lorentz Force Law

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Lorentz Force Law + Newton's Second Law

- Lorentz Force Law: The electromagnetic force on a particle with charge q and velocity $\vec{v}$ produced by electric field $\vec{E}$ and magnetic field $\vec{B}$ is

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}).$$

- If the electromagnetic force is the only force on the particle, then

$$m\frac{d^2\vec{r}}{dt^2} = q\left[\vec{E}(\vec{r}, t) + \frac{d\vec{r}}{dt} \times \vec{B}(\vec{r}, t)\right].$$

- If we know $\vec{E}$ and $\vec{B}$, this is a second-order differential equation we can solve for the vector function $\vec{r}(t)$ that gives the position of the particle.