Kirchhoff's Law

Scott N. Walck

October 1, 2021

Modern Electromagnetic Theory

- The Maxwell Equations

$$\vec{\nabla} \times \vec{B} - \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J} \qquad \vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$$

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$

- The Lorentz Force Law

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Kirchhoff's Law

- Start with Faraday's law.

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

- In static situations, nothing is changing in time, so we can ditch the term with the time derivative. This produces *Kirchhoff's law*.

$$\vec{\nabla} \times \vec{E} = 0$$

- A vector field with zero curl is called *conservative*.
- The electric field is conservative (in static situations).

Conservative fields

- The curl of a field $\vec{E}$ is zero,

$$\vec{\nabla} \times \vec{E} = 0.$$

$$\Updownarrow$$

- The line integral of $\vec{E}$ from $\vec{a}$ to $\vec{b}$ is independent of path.

$$\Updownarrow$$

- The line integral of $\vec{E}$ around any closed path C is zero,

$$\oint_C \vec{E} \cdot d\vec{\ell} = 0.$$

- These three conditions are equivalent. Any field that satisfies one (and hence all) of them is called a *conservative field*.

Coulomb's law is the solution to Gauss's law and Kirchhoff's law.

- If

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0}\rho \qquad\qquad \vec{\nabla} \times \vec{E} = 0$$

- then

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(\vec{r'})}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, d\tau' = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(\vec{r'})}{\mathscr{r}^3}\vec{\boldsymbol{\mathscr{r}}}\, d\tau'$$

$$= \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(\vec{r'})(\vec{r} - \vec{r'})}{\left|\vec{r} - \vec{r'}\right|^3}\, d\tau'.$$