Kirchhoff's Law

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October 1, 2021

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Modern Electromagnetic Theory

The Maxwell Equations

$$\vec{\nabla} \times \vec{B} - \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J} \qquad \qquad \vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$$
$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \qquad \qquad \vec{\nabla} \cdot \vec{B} = 0$$

The Lorentz Force Law

$$ec{F} = q(ec{E} + ec{v} imes ec{B})$$

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Kirchhoff's Law

Start with Faraday's law.

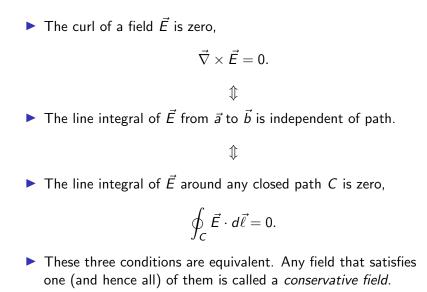
$$\vec{\nabla} \times \vec{E} + rac{\partial \vec{B}}{\partial t} = 0$$

In static situations, nothing is changing in time, so we can ditch the term with the time derivative. This produces *Kirchhoff's law*.

$$\vec{
abla} imes \vec{E} = 0$$

- A vector field with zero curl is called *conservative*.
- The electric field is conservative (in static situations).

Conservative fields



Coulomb's law is the solution to Gauss's law and Kirchhoff's law.

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$$
 $\vec{\nabla} \times \vec{E} = 0$

► If

$$\begin{split} \vec{E}(\vec{r}) &= \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r'})}{2^2} \hat{\boldsymbol{z}} d\tau' = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r'})}{2^3} \vec{\boldsymbol{z}} d\tau' \\ &= \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r'})(\vec{r}-\vec{r'})}{\left|\vec{r}-\vec{r'}\right|^3} d\tau'. \end{split}$$

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