Electrostatic Energy

Scott N. Walck

October 15, 2021

Two kinds of potential energy in mechanics

- Gravitational potential energy

$$\mathrm{PE} = mgy = mgh$$

- Elastic potential energy (spring potential energy)

$$\mathrm{PE} = \frac{1}{2}kx^2$$

In both cases, force points in the direction of decreasing potential energy.

Electric potential energy in old electricity theory

$$\mathrm{PE} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r}$$

- Two positive charges have more potential energy when they are close together than when they are far apart.
- Two negative charges have more potential energy when they are close together than when they are far apart.
- Unlike charges have less potential energy when they are close together than when they are far apart.

Force points in the direction of decreasing potential energy.

Conservative and nonconservative forces

Force	Conservative?	Potential Energy
Gravity	Conservative	$\mathrm{PE} = mgy$
Normal Force	Nonconservative	none
Rope Tension	Nonconservative	none
Friction	Nonconservative	none
Spring	Conservative	$\mathrm{PE} = \frac{1}{2}kx^2$
Electric	Conservative	$\mathrm{PE} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r}$

Electric potential energy in the new electromagnetic theory

- Electric Potential $\neq$ Electric Potential Energy
- Aspect 1: Charge creates electric potential.
- Aspect 2: Electric potential invests charge with potential energy.

What is Electric Potential?

- Electric potential is something that permeates all space.
- Each point in space has a number for electric potential.
- Electric potential is a scalar field.
- Electric charge produces electric potential.
- Electric potential invests charge with potential energy.
- Electric potential can change in time, so it permeates all space-time.
- Electric field points in the direction of decreasing electric potential.
- Symbol: V
- SI Unit: Volt (V)

Electric potential is produced by charge.

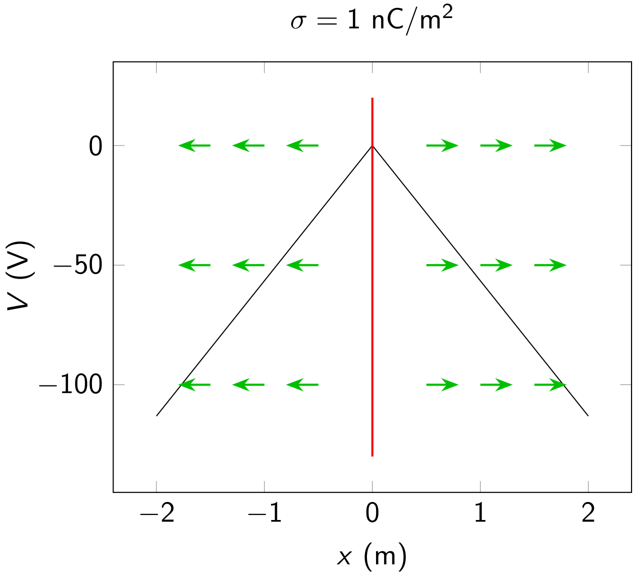
$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{\left|\vec{r}-\vec{r}_i'\right|}$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_C \frac{\lambda(\vec{r}')}{\mathscr{r}}\,d\ell' = \frac{1}{4\pi\epsilon_0}\int_C \frac{\lambda(\vec{r}')}{\left|\vec{r}-\vec{r}'\right|}\,d\ell'$$

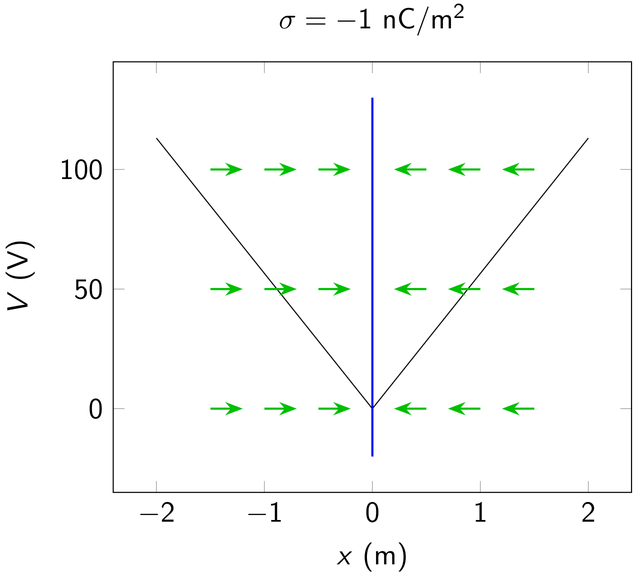
$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_S \frac{\sigma(\vec{r}')}{\mathscr{r}}\,da' = \frac{1}{4\pi\epsilon_0}\int_S \frac{\sigma(\vec{r}')}{\left|\vec{r}-\vec{r}'\right|}\,da'$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(\vec{r}')}{\mathscr{r}}\,d\tau' = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(\vec{r}')}{\left|\vec{r}-\vec{r}'\right|}\,d\tau'$$

Electric potential produced by a positively charged plate



Electric potential produced by a negatively charged plate



Electric potential produced by a charged plate

The electric potential produced by a plate with surface charge density σ at a distance z from the plate is

$$V = -\frac{\sigma}{2\epsilon_0} |z| .$$

Electric potential produced by two plates

- To find the electric potential produced by two or more plates, add the electric potential produced by each plate alone.
- Electric potential is a scalar, so you only need to add numbers, not vectors.

$$V(z) = -\frac{\sigma_1}{2\epsilon_0}\left|z - z_1\right| - \frac{\sigma_2}{2\epsilon_0}\left|z - z_2\right|$$

Superposition Principle for Electric Potential

- The electric potential produced by multiple objects is the (scalar) sum of the electric potentials produced by each object alone.
- Uses for superposition:
  - Electric potential produced by two plates
  - Electric potential produced by a particle and a plate
  - Electric potential produced by two particles

Electric potential energy in the new electromagnetic theory

- Aspect 2: Electric potential invests charge with potential energy.

$$W = QV(\vec{r})$$

- This W is the (minimum) work required to move point charge Q from infinity (where the electric potential is zero) to position $\vec{r}$. It does not matter what path we take.

Whose energy is it?

In the old Coulumb theory, a system of two point charges, q and Q, spaced a distance $\mathscr{r}$ apart, has a potential energy

$$\mathrm{PE} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{\mathscr{r}}.$$

In the old Coluomb theory, it is the *system* of charged particles that claims ownership of the energy.

Three expressions for energy

- For point particles, we can view energy as belonging to the system of point particles:

$$W = \frac{1}{2}\sum_{i=1}^{n} q_i V(\vec{r}_i)$$

- For charge density ρ, we can view energy as belonging to the charge:

$$W = \frac{1}{2}\int \rho V \, d\tau$$

- In modern Faraday-Maxwell electromagnetic theory, we are invited to view energy as being owned by the electric field:

$$W = \frac{\epsilon_0}{2}\int E^2 \, d\tau$$