Electromotance

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November 17, 2021

Emf is the ambulation of a force per unit charge.

- An *ambulation* is a dotted line integral over a curve.
- If $\vec{f}$ is a force per unit charge, and C is a curve, then

$$\mathcal{E} = \int_C \vec{f} \cdot d\vec{\ell}$$

  is the emf of that force per unit charge over the curve C.
- $\vec{E}$ is a force per unit charge.
- $\vec{v} \times \vec{B}$ is a force per unit charge.
- We use the symbol $\mathcal{E}$ for emf.
- The SI unit for emf is the volt.

Electrical emf

- An electrical emf over a (closed or open) curve C is

$$\mathcal{E} = \int_C \vec{E} \cdot d\vec{\ell}$$

- If $\vec{E}$ is an electrostatic field and C is a closed curve, then the electric emf must be zero, because electrostatic fields are conservative.
- In electrodynamics, the electrical emf around a closed curve can be nonzero.

Magnetic emf

- A magnetic emf over a (closed or open) curve C is

$$\mathcal{E} = \int_C (\vec{v} \times \vec{B}) \cdot d\vec{\ell}$$

- Simpest situation:
  - $\vec{B}$ is uniform
  - $\vec{v} \perp \vec{B}$
  - $\vec{v} \times \vec{B}$ is in the same direction as $d\vec{\ell}$
  - $\mathcal{E} = vB\ell$
- Griffiths calls this *motional emf*. It's what you get when a wire or metal bar is moving through a magnetic field.

Flux

- The *flux* of a vector field $\vec{F}$ over a surface S is defined to be the dotted surface integral (or flux integral) of $\vec{F}$ over S.

$$\int_S \vec{F} \cdot d\vec{a}$$

- If the vector field is the magnetic field, we call the flux *magnetic flux*.

$$\Phi_B = \int_S \vec{B} \cdot d\vec{a}$$

- The surface S can be open or closed, but you must have a surface to talk about magnetic flux.

Integral form of the no-magnetic-monopoles law

- Start with the differential form of the no-magnetic-monopoles law (sometimes called Gauss's law for magnetism).

$$\vec{\nabla} \cdot \vec{B} = 0$$

- Pick any volume V. Integrate both sides of the no-magnetic-monopoles law over this volume V.

$$\int_V (\vec{\nabla} \cdot \vec{B}) \, d\tau = 0$$

- Use the divergence theorem to rewrite the left side.

$$\int_{\partial V} \vec{B} \cdot d\vec{a} = 0$$

- The magnetic flux through any closed surface is zero.

Flux Rule

Let S be any surface. The emf around the boundary of the surface is related to the rate of change of magnetic flux through the surface.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- The emf around the boundary of a surface is the negative rate of change of magnetic flux through the surface.