

EM Statics

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August 29, 2025

Modern Electromagnetic Theory

► The Maxwell Equations

$$\begin{aligned}\vec{\nabla} \times \vec{B} - \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} &= \mu_0 \vec{J} & \vec{\nabla} \cdot \vec{E} &= \frac{1}{\epsilon_0} \rho \\ \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} &= 0 & \vec{\nabla} \cdot \vec{B} &= 0\end{aligned}$$

► The Lorentz Force Law

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

The Maxwell equations decouple in the static case.

$$\begin{aligned}\vec{\nabla} \times \vec{B} - \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} &= \mu_0 \vec{J} & \vec{\nabla} \cdot \vec{E} &= \frac{1}{\epsilon_0} \rho \\ \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} &= 0 & \vec{\nabla} \cdot \vec{B} &= 0\end{aligned}$$

- By “the static case”, I mean none of ρ , \vec{J} , \vec{E} , or \vec{B} changes in time.

$$\begin{aligned}\vec{\nabla} \cdot \vec{E} &= \frac{1}{\epsilon_0} \rho & \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} &= 0 & \vec{\nabla} \times \vec{B} &= \mu_0 \vec{J}\end{aligned}$$

Static Maxwell equations

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$$

$$\vec{\nabla} \times \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

- ▶ Left two equations describe electrostatics.
- ▶ Right two equations describe magnetostatics.
- ▶ In the static case, \vec{E} and \vec{B} do not appear together in any Maxwell equation.
- ▶ In the static case, electricity and magnetism are separate subjects.