Maxwell's Equations and Differential Vector Calculus

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Modern Electromagnetic Theory

- The Maxwell Equations

$$\vec{\nabla} \times \vec{\mathbf{B}} - \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} = \mu_0 \vec{\mathbf{J}} \qquad \vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_0} \rho$$

$$\vec{\nabla} \times \vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{B}}}{\partial t} = 0 \qquad \vec{\nabla} \cdot \vec{\mathbf{B}} = 0$$

- The Lorentz Force Law

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

A Backwards Problem

What charge distribution gives rise to the following electric field?

$$\vec{\mathbf{E}} = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}\hat{\mathbf{r}} & , \quad r \leq R \\ \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} & , \quad r > R \end{cases}$$