Vectors

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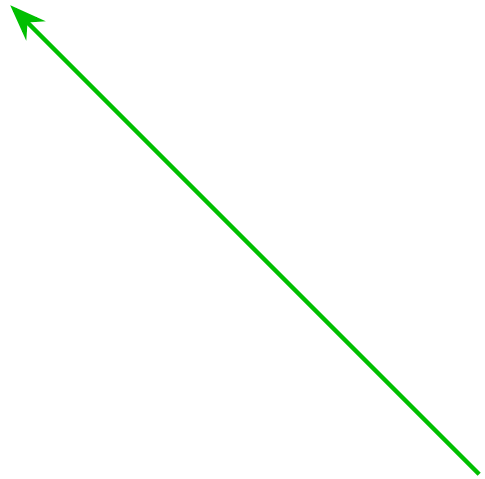
Vectors in 3-dimensional space

- Picture a vector as an arrow.
- By an arrow, we mean a straight line segment with an arrow head on one end.
- A vector has a magnitude and (almost always) a direction.

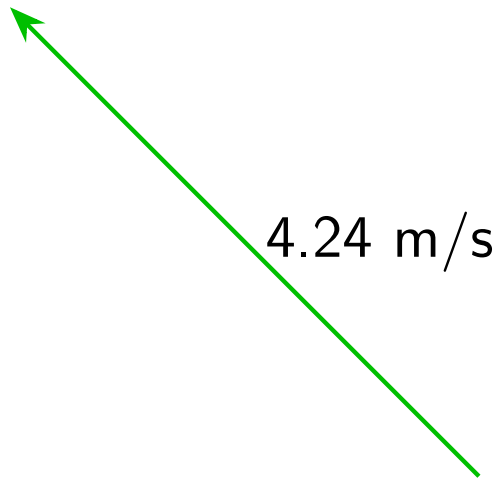
The *direction* of the vector points with the arrowhead.

- A vector pointing northwest



The *magnitude* of the vector is the length of the arrow.

- A vector with magnitude 4.24 m/s.



There is a *zero vector*.

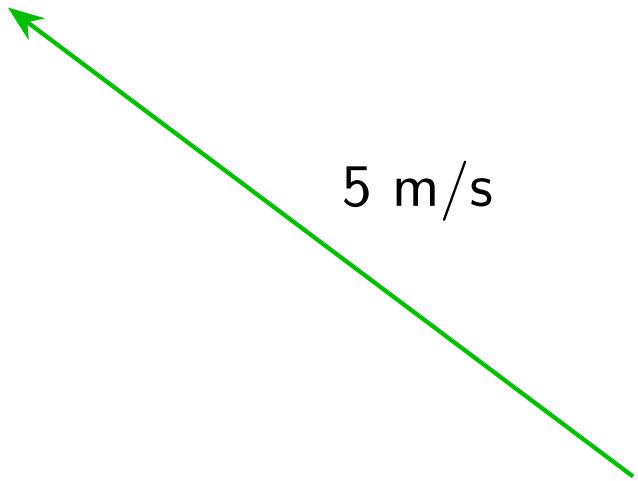
- The magnitude of the zero vector is zero.
- If a vector has a magnitude of zero, it must be the zero vector.

$$\text{zero vector} \Leftrightarrow \text{vector with zero magnitude}$$

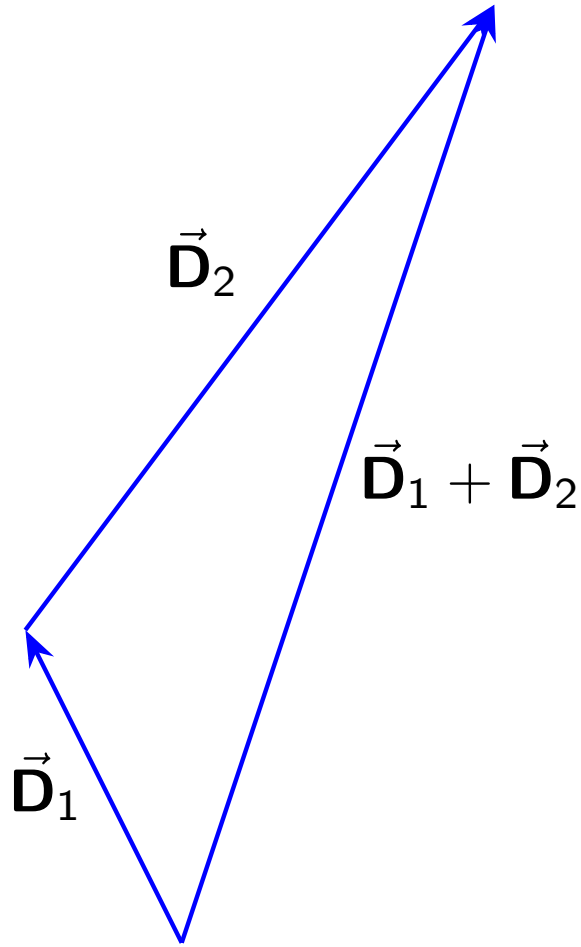
- The zero vector does not have a direction. (It's the only vector that has no direction.)
- We can draw the zero vector as a point, or not draw it at all.
- If an object is not moving, its velocity is represented by the zero vector.
- If an object is not accelerating, its acceleration is represented by the zero vector.

All nonzero vectors have a magnitude and a direction.

- The magnitude of a nonzero vector is a positive number, along with units.
- We draw a vector as an arrow.
- The length of the arrow represents the magnitude of the vector.
- The direction of the arrow represents the direction of the vector.

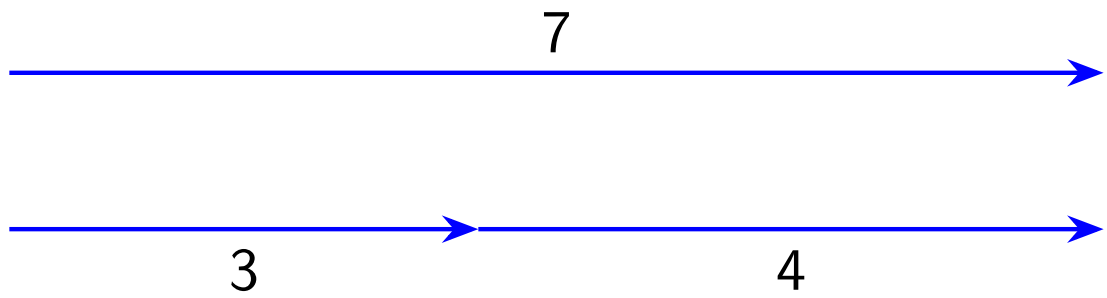


To add vectors, put them tip-to-tail

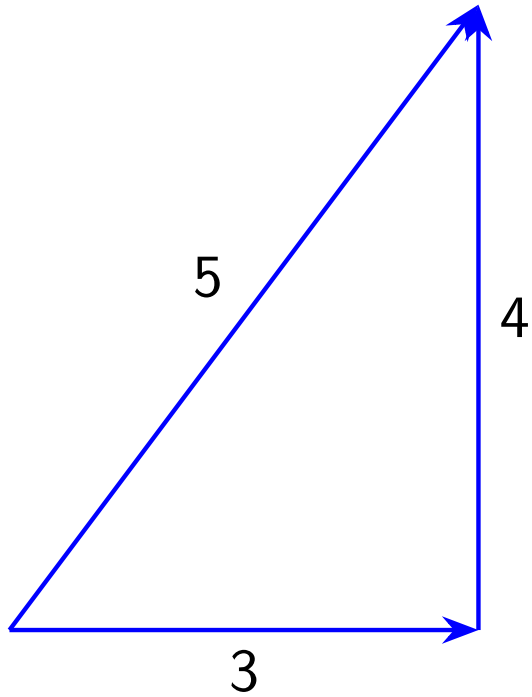


- Vector sum points from tail of first vector to tip of last vector

Vectors can do everything that numbers can do

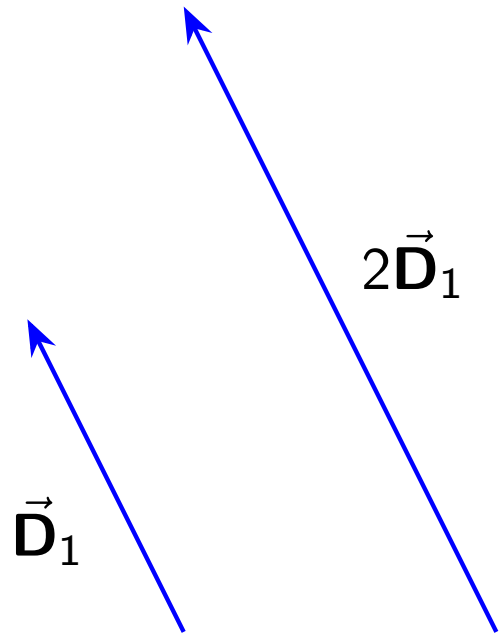


Vectors can do more than numbers can do

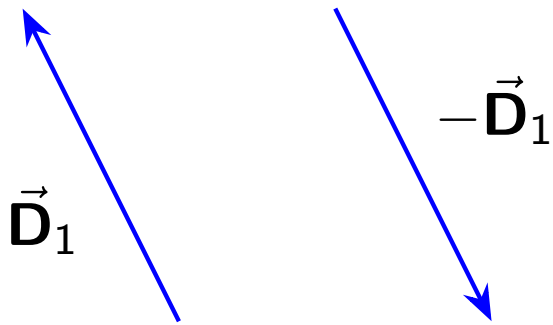


- A vector with magnitude 3 plus a vector with magnitude 4 can produce a vector with magnitude 5.
- Magnitude of the sum (5) $\neq$ the sum of the magnitudes (7).

Vectors can scale

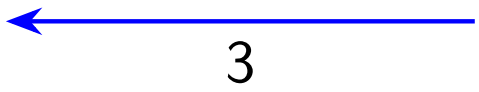


To negate a vector, flip its direction

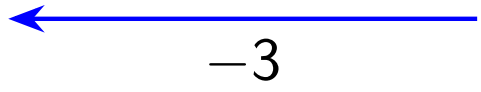


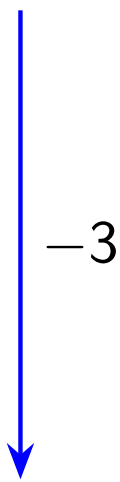
Vectors have magnitude and direction

Write this



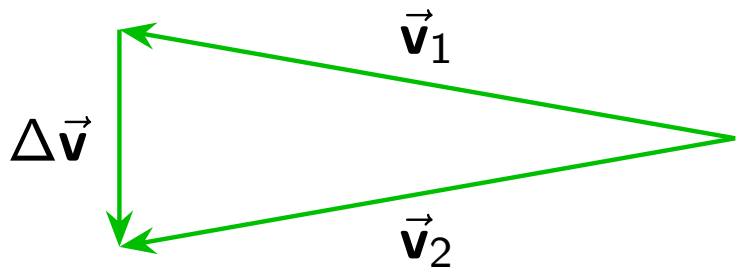
Don't write this





- You can label a vector with its magnitude.
- The magnitude of a vector is never negative.

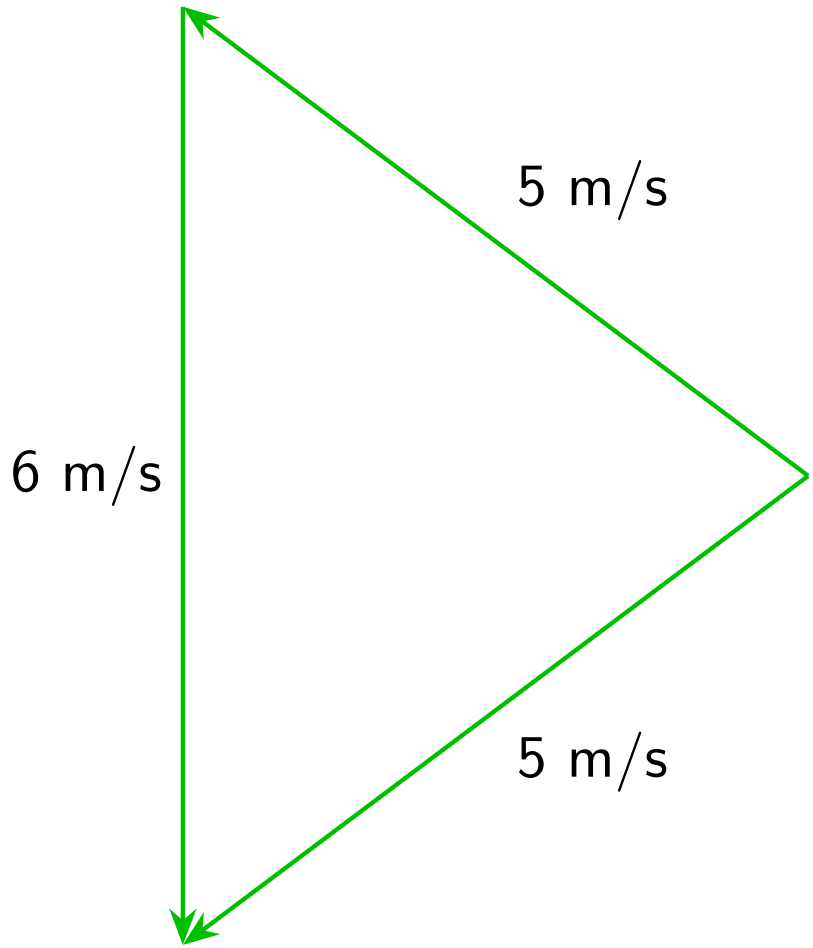
To subtract vectors, put them tail-to-tail



$$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1$$

$$\vec{\mathbf{v}}_2 = \vec{\mathbf{v}}_1 + \Delta\vec{\mathbf{v}}$$

Magnitude of difference $\neq$ difference of magnitudes



- Magnitude of difference $= 6$ m/s
- Difference of magnitudes $= 0$ m/s

Vectors vs. Numbers

- Numbers can be positive.
- Numbers can be negative.
- Numbers can increase (over time).
- Numbers can decrease (over time).
- Vectors cannot be positive.
- Vectors cannot be negative.
- Vectors cannot increase (over time).
- Vectors cannot decrease (over time).
- Numbers have an order. If a and b are numbers on the number line, then either $a > b$, $a < b$, or $a = b$.
- Vectors do not have an order.