Vectors

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What is a vector?

Mathematicians and physicists don't entirely agree.

- Mathematician: "A vector is an element of a vector space."
- Physicist: "A vector is any set of three components that transforms in the same manner as a displacement when you change coordinates." (Griffiths)

Physicist's vector vs. mathematician's vector.

	Physicist	Mathematician
number of components	3	n
coordinates	must be present	not needed

Does the vector refer to physical space?

Is

$$\begin{bmatrix} 3 \text{ apples} \\ 2 \text{ pears} \\ 7 \text{ bananas} \end{bmatrix}$$

a vector?

- Mathematician: "Sure. Why not?"
- Physicist: "No. There is no notion of coordinates in 3-dimensional space here. It's not a vector."

Components of a vector

- Physicist: "Given a coordinate system, you can talk about components of a vector."
- Mathematician: "Given a basis, you can talk about components of a vector. The basis could come from a coordinate system, but it doesn't need to."

Differences in notation

- Physicists like to put an arrow over a vector when writing it, or display it in boldface type.
  - Vectors: $\mathbf{v}$, $\vec{v}$, $\vec{\mathbf{v}}$
  - Scalar: v
- Mathematicians have no need for these pleasantries.
- A common convention in physics is for a non-boldface symbol to stand for the magnitude of the associated boldface vector.
  - If $\vec{\mathbf{v}}$ is a velocity, then $v = |\vec{\mathbf{v}}|$ is the speed associated with that velocity.

  If this convention is in force, it's really important whether the arrow/boldface is present or not, because the symbol with the arrow means something different from the symbol without the arrow.

Mathematicians and physicists don't entirely disagree.

- Everyone agrees that vectors in 2 and 3 dimensions are important. They are also the easiest to visualize.
- Everyone agrees that vectors can be added and scaled.

We will focus on the wide realm of agreement between mathematicians and physicists.

- Physicists like to use $\hat{\mathbf{i}}$ to represent a unit vector in the x direction.
- Similarly, $\hat{\mathbf{j}}$ is a unit vector in the y direction.
- $\hat{\mathbf{k}}$ is a unit vector in the z direction.