Calculus

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What is Calculus?

Calculus is the mathematics of change.

What is change?

- We are interested in how things change in time
  - The temperature on my back porch changes as summer ends and fall begins.
  - The speed of my car changes as I approach a stop sign.
  - When something changes in time, we call time the independent variable.
- The independent variable does not need to be time.
  - The kinetic energy of an object changes with its speed.

$$\mathrm{KE} = \frac{1}{2}mv^2$$

    In this case, speed is the independent variable.
  - The kinetic energy of an object changes with its momentum.

$$\mathrm{KE} = \frac{p^2}{2m}$$

    In this case, momentum is the independent variable.

What is changing, and what is it changing with?

- The thing that is changing is called the dependent variable. It depends on some other thing.
- The other thing is called the independent variable.
- The dependent variable is a function of the independent variable.
- In Calculus I, the independent variable is a number and the dependent variable is a number. For example, the independent variable could be time and the dependent variable could be the temperature on my back porch. Both of these are numbers.
- In Calculus III, either the independent variable, or the dependent variable, or both are something more complex than a number, like a vector.

How can we describe change?

- To talk about change, we need a dependent variable that depends on an independent variable.
- We say that the dependent variable is a *function* of the independent variable.
- Mathematical functions are central to calculus.