Conservation of Linear Momentum: the Ballistic Pendulum

I. Discussion

a. Determination of Velocity from Collision

In the typical use made of a ballistic pendulum, a projectile, having a small mass, m, and a horizontal velocity, v, strikes and imbeds itself in a pendulum bob, having a large mass, M, and an initial horizontal velocity of zero. Immediately after the collision occurs the combined mass, M + m, of the bob and projectile has a small horizontal velocity, V.

During this collision the conservation of linear momentum holds. If rotational effects are ignored, and if it is assumed that the time required for this event to take place is too short to allow the projectile to be substantially influenced by the earth's gravitational field, the motion remains horizontal, and

$$mv = (M + m)V \qquad (1)$$

After the collision has occurred, the combined mass, (M + m), rises along a circular arc to a vertical height h in the earth's gravitational field, as shown in Fig. I (c). During this portion of the motion, but not during the collision itself, the conservation of energy holds, if it is assumed that the effects of friction are negligible. Then, for motion along the arc,

$$\tfrac{1}{2}(M + m)V^2 = (M + m)gh \quad \text{, or} \qquad (2)$$

$$V = \sqrt{2gh} \qquad (3)$$

When V is eliminated using equations 1 and 3, the velocity of the projectile is

$$v = \frac{M + m}{m}\sqrt{2gh} \qquad (4)$$

b. Determination of Velocity from Range

When a projectile, having an initial horizontal velocity, v, is fired from a height H above a plane, its horizontal range, R, is given by

$$R = v_{xo} t \qquad (5)$$

and the height H is given by

$$H = \frac{1}{2} g t^2 \qquad (6)$$

where t = time during which projectile is in flight.

When the time of flight, t, is eliminated between equations (5) and (6), the result yields the initial horizontal velocity of the projectile

$$v = \frac{R}{\sqrt{\frac{2H}{g}}} \qquad (7)$$

II. Collection of Data

a. Determination of Velocity from Collision

In this experiment, the projectile, a metal ball, is fired by a spring-gun into a hollow pendulum bob. After rising, the pendulum is held at its highest point by a pawl which engages a toothed sector. A small pointer is located on the pendulum bob to mark the position of the center of mass of the combined ball and bob. If the height h1, of the bob before rising, and the height h2, of the bob after rising, are measured with respect to the base of the pendulum, the rise of the center of mass is given by

$$h = h2 - h1 \qquad (8)$$

Make ten firings of the ball into the pendulum, and record the number N, corresponding to the rest position of the bob after each firing. Determine the average value of these numbers, and set the bob in the position corresponding to this average. Then measure the value of h2 , corresponding to this average angular position. Measure h1 when the pendulum is hanging in its lowest position. Remove the pendulum and determine its mass, M, and the mass, m, of the ball.

b. Determination of Velocity from Range

Make about ten firings of the ball from the laboratory table to the floor, while the pendulum is removed. In order to record the range R, it is necessary to staple a piece of carbon paper between two pieces of ordinary paper. (The carbon paper is usually so fragile that the ball tears it upon impact.) The range for the spring-guns used in this experiment is about 2 to 3 meters, although a trial firing should be made before introducing the carbon paper record sheet. Tape this record sheet to the floor. The firings are then made onto this sheet. Measure the distances, D, from a plumb-line, which passes through the center of the ball. Finally, measure the distance, d, to the firing marks from the front edge of the paper. Determine the average of these ten distances and

$$R = D + d_{ave.} \qquad (9)$$

Use the provided plumb bob and level and find a good way to measure D and H. Remember, H is the vertical distance from the <u>bottom</u> of the ball to the floor and D is the horizontal distance from the ball position where it loses contact with the gun to the paper sheet.

III. Calculations and results.

From the collision method (eqn 4) calculate the ball's initial velocity and from the range method (eqn 7) calculate the ball's initial velocity. Compute the percent difference between the two methods. Comment on any discrepancy and discuss uncertainties.

IV. Question.

Is the collision between the ball and the pendulum elastic or inelastic?

Justify your answer by calculating the kinetic energy of the system (ball and pendulum) before the collision and compare the kinetic energy just after the collision. (Hint: Use m, v to calculate the initial kinetic energy of the ball since the pendulum is initially at rest. Then use M, m, V to find the final kinetic energy or use M, m, h to calculate the potential energy at the highest point which is equal to the kinetic energy of the system after the collision.)