Modern Physics

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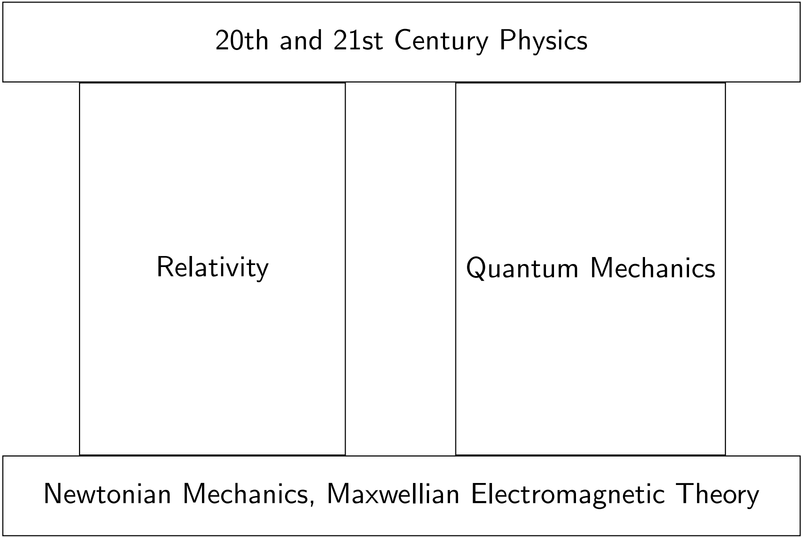
Two Pillars of 20th Century Physics



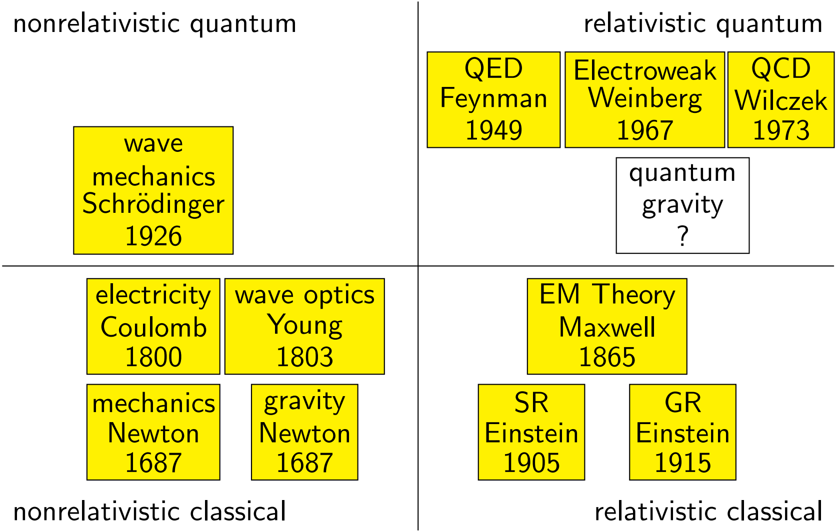
There are two types of relativity theory.

- Special Relativity
  - published in 1905 by Albert Einstein
  - Time is not universal.
  - Each observer has a clock. Different observers have clocks that run at different rates.
  - You can only compare clocks that are next to each other.
  - Three-dimensional space and one-dimensional time belong together in a four-dimensional spacetime.
- General Relativity
  - published in 1915 by Albert Einstein
  - a theory of gravity
  - Gravity is not a force.
  - Our 4-dimensional spacetime in not flat. It is curved.
  - Mass causes spacetime to curve.
  - Objects experience gravity by moving in a “straight line” through the curved spacetime.

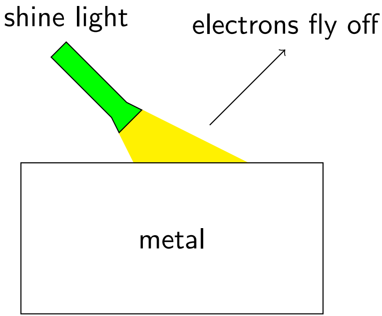
Key events in the development of Quantum Theory

1900	Planck proposes quanta of light
1905	Einstein explains photoelectric effect
1913	Bohr suggests special radii
1921	Stern and Gerlach demonstrate spatial quantization
1923	Compton sees frequency shift in scattered X-rays
1924	de Broglie suggests matter waves
1925	Heisenberg presents matrix mechanics
1926	Schrödinger presents wave mechanics
1927	Heisenberg presents uncertainty principle

Theories in Physics

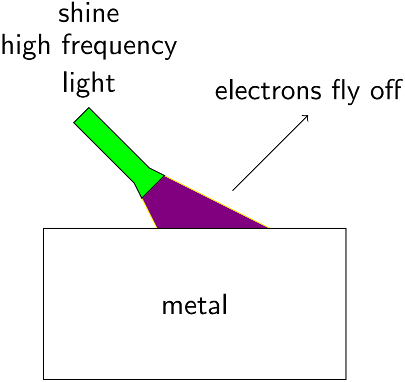


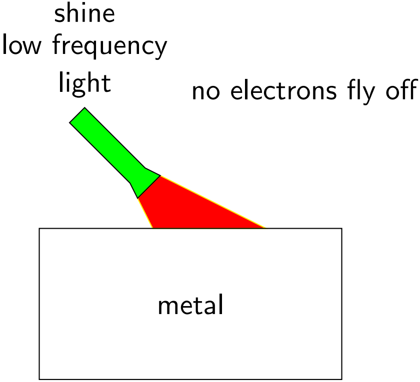
The photoelectric effect



- Light provides the energy needed to free electrons from the metal.

The photoelectric effect

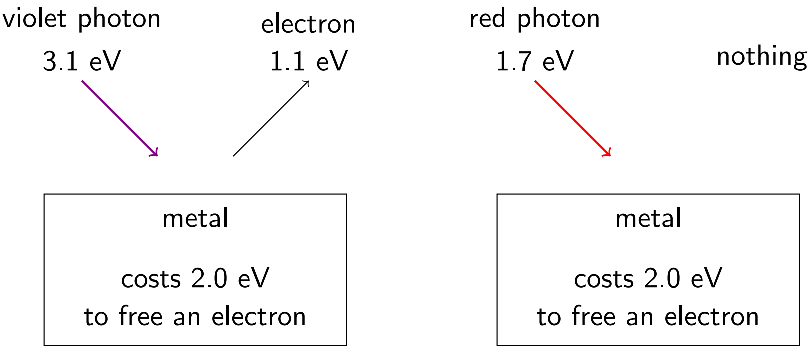




- violet has high frequency
- even if light is very bright

The photoelectric effect: Einstein's idea

- One photon must free one electron.
- Energy of one photon is $E = hf$.



How to convert among photon wavelength, frequency, and energy

Quantity	Symbol	Unit
Wavelength	λ	m, nm
Frequency	f	Hz, THz
Energy	E	J, eV

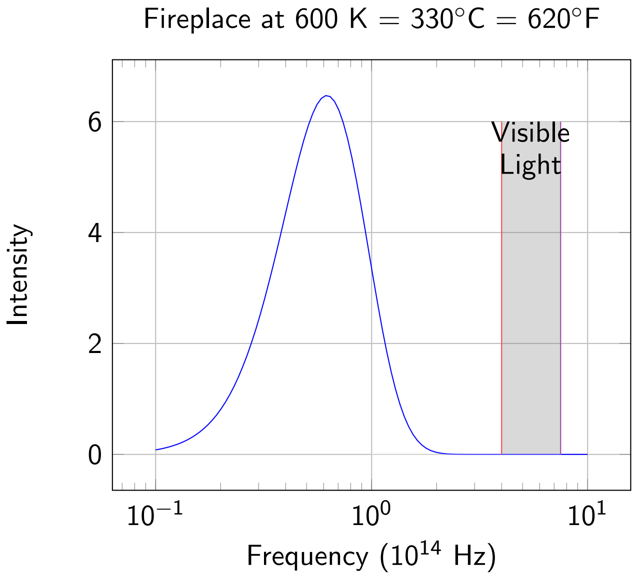
$$f = \frac{c}{\lambda}$$

$$E = hf$$

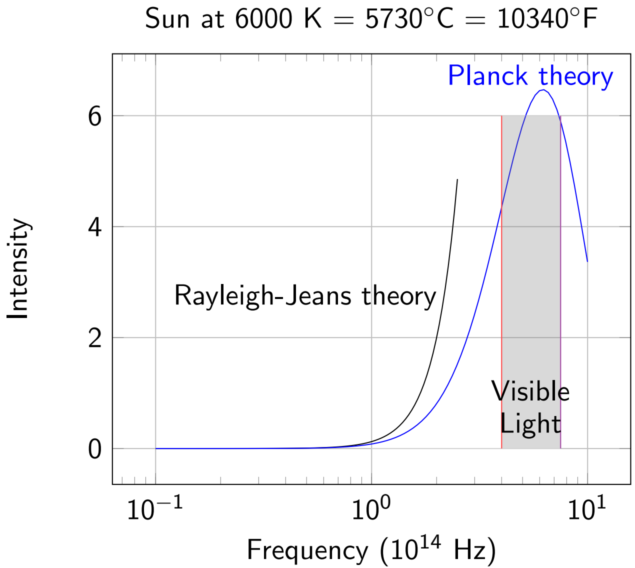
$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.626 \times 10^{-34} \text{ J s} = 4.136 \times 10^{-15} \text{ eV s}$$

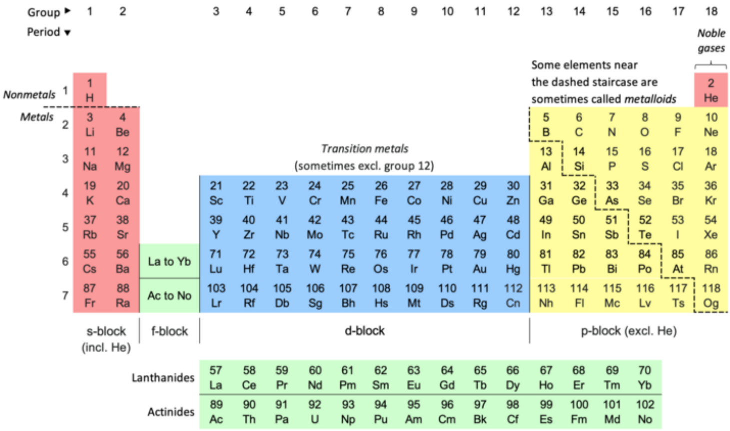
Hot things radiate



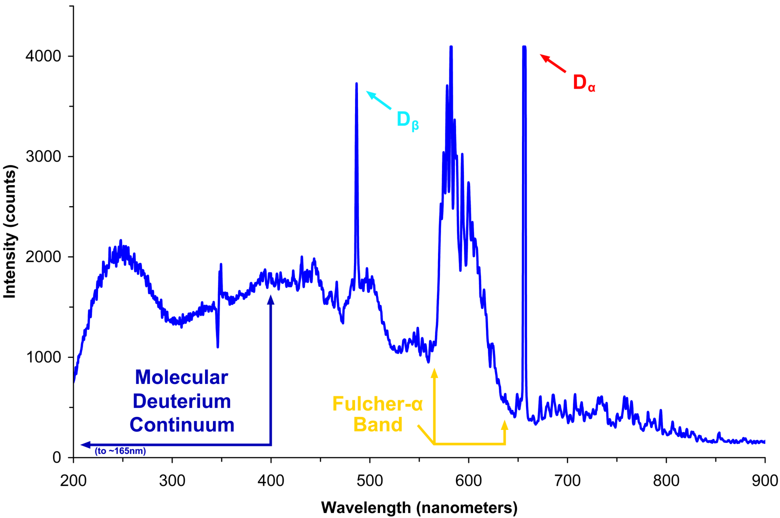
The ultraviolet catastrophe



Physicists love hydrogen.

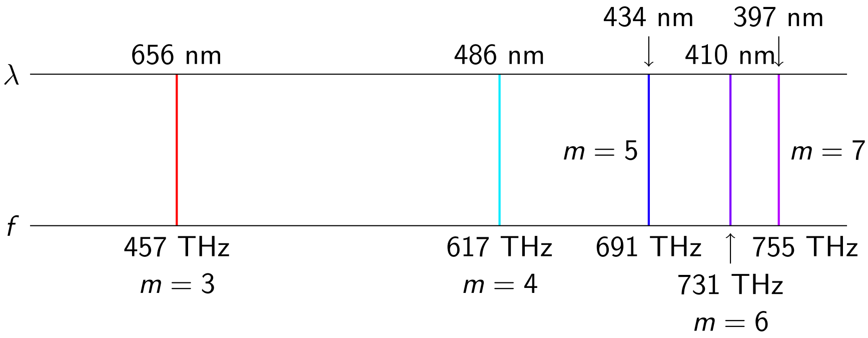


Balmer: Hydrogen spectrum has sharp lines



- unlike a blackbody

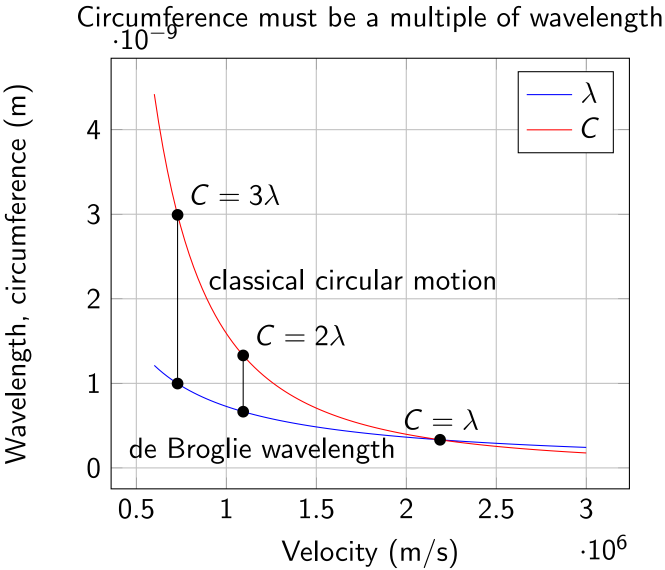
Balmer (1885) looked at 5 lines from hydrogen



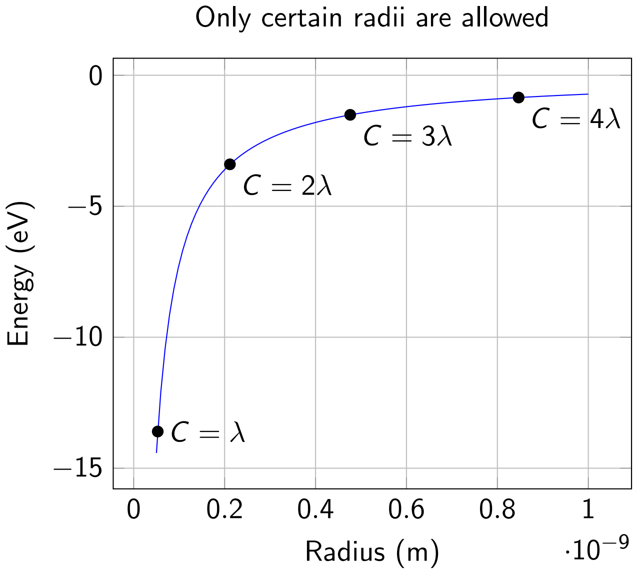
He found a pattern:

$$\lambda = \frac{365\ \mathrm{nm}}{1 - \frac{4}{m^2}} \qquad f = 3289\ \mathrm{THz}\left(\frac{1}{2^2} - \frac{1}{m^2}\right)$$

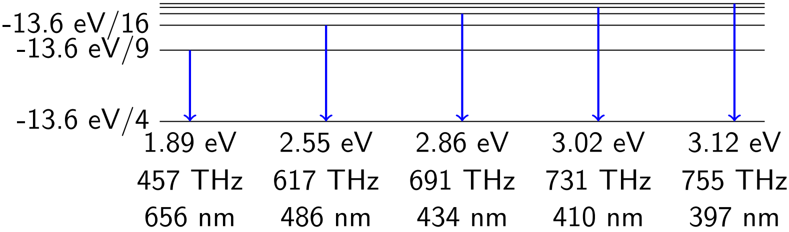
Bohr atom (1913)



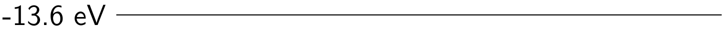
Bohr model produces hydrogen energy levels



Hydrogen energy levels



Differences in energy levels
match the Balmer frequencies



Mass is a form of energy.

- In 1905, Einstein wrote the following equation.

$$E = mc^2$$

- What does it mean? It means that a thing has energy, simply by virtue of having mass. This energy is called *rest energy*.
- An object with mass m has rest energy equal to mc^2.
- Rest energy can be transformed into other types of energy, resulting in a loss of mass.

An MeV/c^2 is a unit of mass.

$$
\begin{aligned}
1\ \mathrm{MeV}/c^2 &= \frac{1\ \mathrm{MeV}}{c^2} = \frac{1\ \mathrm{MeV}}{(3 \times 10^8\ \mathrm{m/s})^2} \times \frac{10^6\ \mathrm{eV}}{1\ \mathrm{MeV}} \\
&= \frac{10^6\ \mathrm{eV}}{(3 \times 10^8\ \mathrm{m/s})^2} \times \frac{1.602 \times 10^{-19}\ \mathrm{kg\ m^2/s^2}}{1\ \mathrm{eV}} \\
&= \frac{1.602 \times 10^{-13}\ \mathrm{kg\ m^2/s^2}}{9 \times 10^{16}\ \mathrm{m^2/s^2}} \\
&= 1.78 \times 10^{-30}\ \mathrm{kg}
\end{aligned}
$$

The mass of a hydrogen atom is slightly less than the sum of the masses of a proton and an electron.

proton	938.2720133 MeV/c^2
electron	0.5109989 MeV/c^2
total	938.7830122 MeV/c^2

proton + electron	938.7830122 MeV/c^2
hydrogen	938.7829986 MeV/c^2
difference	0.0000136 MeV/c^2

$$0.0000136\ \mathrm{MeV}/c^2 = 13.6\ \mathrm{eV}/c^2$$

Some particles are stable and some are not

- Stable particles
  - Proton
  - Electron
  - Helium atom
  - Many atoms
- Unstable particles
  - Neutron
  - Radium-226
  - Uranium-232
  - Carbon-14

Unstable particles spontaneously decay.

There are three main mechanisms by which nuclei decay.

- Alpha decay (α decay)
  - Emission of a helium-4 nucleus
- Beta decay (β decay)
  - Emission of an electron
- Gamma decay (γ decay)
  - Emission of a photon

In alpha decay, a nucleus splits into a helium-4 nucleus and something else.

- $^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}$
- $^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$
- $^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He}$
- An alpha particle is a $^{4}_{2}\text{He}$ nucleus.

Beta decay involves an electron or related particle.

- β^- decay involves emission of an electron.
- β^+ decay involves emission of a positron.
- Electron capture involves absorption of an electron.

In β^- decay, a neutron becomes a proton, an electron, and an antineutrino.

- $n \rightarrow \mathrm{p} + \mathrm{e}^- + \overline{\nu}$
- ${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \mathrm{e}^- + \overline{\nu}$
- ${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X} \rightarrow {}^{\mathrm{A}}_{\mathrm{Z+1}}\mathrm{Y} + \mathrm{e}^- + \overline{\nu}$
- Charge is conserved.
- The number of protons is not conserved.
- A beta particle is an electron.

β^+ decay involves emission of a positron.

- ${}^{19}_{10}\mathrm{Ne} \rightarrow {}^{19}_{9}\mathrm{F} + \mathrm{e}^+ + \nu$
- ${}^{18}_{9}\mathrm{F} \rightarrow {}^{18}_{8}\mathrm{O} + \mathrm{e}^+ + \nu$ (PET scan)
- ${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X} \rightarrow {}^{\mathrm{A}}_{\mathrm{Z}-1}\mathrm{Y} + \mathrm{e}^+ + \nu$
- Behind the scenes, $p \rightarrow \mathrm{n} + \mathrm{e}^+ + \nu$, although this reaction itself never occurs.
- Charge is conserved.
- The number of protons is not conserved.

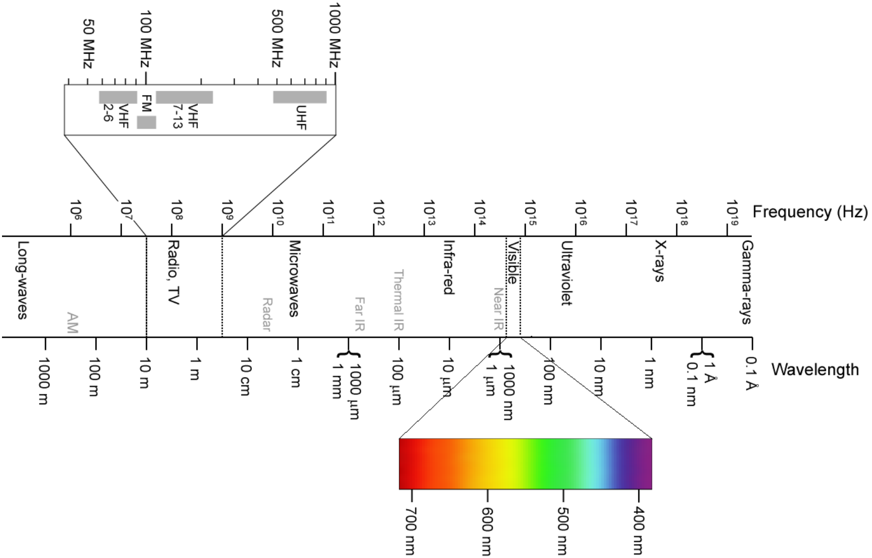
Electron capture involves absorption of an electron.

- ${}^{7}_{4}\mathrm{Be} + \mathrm{e}^- \rightarrow {}^{7}_{3}\mathrm{Li} + \nu$
- ${}^{18}_{9}\mathrm{F} + \mathrm{e}^- \rightarrow {}^{18}_{8}\mathrm{O} + \nu$
- ${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X} + \mathrm{e}^- \rightarrow {}^{\mathrm{A}}_{\mathrm{Z}-1}\mathrm{Y} + \nu$
- Behind the scenes, $\mathrm{p} + \mathrm{e}^- \rightarrow \mathrm{n} + \nu$.
- Charge is conserved.
- The number of protons is not conserved.

Gamma decay involves the emission of a photon.

- ${}^{12}_{6}\mathrm{C}^* \rightarrow {}^{12}_{6}\mathrm{C} + \gamma$
- ${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}^* \rightarrow {}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X} + \gamma$
- Charge is conserved.
- The number of protons is conserved.
- Similar to the emission of a photon from an excited state of hydrogen, except that in hydrogen, an excited *electronic* state makes a transition to a lower energy level, while here an excited *nuclear* state makes a transition to a lower energy level.
- A gamma particle is a photon.

Gamma rays are the highest energy photons.



Isotopes of Carbon

Isotope	Abundance	Half-life
^{11}C	0	20.334 min
^{12}C	99%	stable
^{13}C	1%	stable
^{14}C	1.3×10^{-12}	5730 years

Earth's atmosphere contains carbon-14.

- We want to apply ideas of nuclear decay to the problem of figuring out how old something is.

$$N = N_0 e^{-\lambda t}$$

- Carbon-14 is unstable and decays, but it is also produced by the following reaction.

$$\mathrm{n} + {}^{14}_{7}\mathrm{N} \rightarrow {}^{14}_{6}\mathrm{C} + \mathrm{p}$$

- The fraction of ^{14}C in Earth's atmosphere has remained roughly constant over thousands of years.

$$\frac{N(^{14}\mathrm{C})}{N(^{12}\mathrm{C})} = 1.3 \times 10^{-12}$$

Carbon Dating

- Every living thing maintains the ratio of 1.3×10^{-12} carbon-14 atoms for each carbon-12 atom by interacting with Earth's atmosphere (by breathing, inspiration of CO_2, etc.)
- When a living thing dies, this interaction stops, no new ^{14}C comes in, and the existing ^{14}C decays.
- By knowing the initial fraction of carbon-14 is 1.3×10^{-12} and measuring the present fraction, we can tell how long it has been since something died.

To find the time since something has been alive:

$$N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$t = -\frac{\ln\left(\frac{N}{N_0}\right)}{\lambda}$$