Lorentz Force Law

Scott N. Walck

February 5, 2021

Electric field exerts force on charge

- Lorentz force law: The force on a particle with charge q sitting in an electric field $\vec{\mathsf{E}}$ is

$$\vec{\mathsf{F}} = q\vec{\mathsf{E}}$$

  where $\vec{\mathsf{E}}$ is evaluated at the position of the particle.
- If the particle has positive charge, the force on it points in the same direction as the electric field (at the position of the particle).
- If the particle has negative charge, the force on it points opposite the electric field.
- For magnitudes:

$$F = |q|\, E$$

If you have a coordinate system:

Since the Lorentz force law,

$$\vec{F} = q\vec{E},$$

is a vector equation, we can write an equation for each component of the vectors.

$$F_x = qE_x$$
$$F_y = qE_y$$
$$F_z = qE_z$$

Two aspects to electromagnetic theory

- Aspect 1: Charge creates electric field.

$$E = k\frac{|Q|}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{|Q|}{r^2}$$

  Sometimes I call this equation Coulomb's law, although that can be confusing since the force equation with two particles is the original Coulomb's law.

  - Electric field points away from positive charge.
  - Electric field points toward negative charge.

- Aspect 2: Electric field exerts force on charge.

$$\vec{F} = q\vec{E}$$

  The Lorentz force law expresses aspect 2.

Combine the two aspects and we arrive back at the original Coulomb's law.

If we first calculate the electric field $\vec{E}$ produced by a particle with charge Q,

$$E = k\frac{|Q|}{r^2}$$

and then apply the Lorentz force law to find the force on a particle with charge q a distance r away from Q,

$$F = |q|\,E = k\,|q|\,\frac{|Q|}{r^2} = k\frac{|qQ|}{r^2},$$

we recover Coulomb's 18th century electricity law.

- Comforting, because ideas are fitting together.
- Concerning, because why do we bother with electric field?

Why do we bother with electric field?

- Electric field offers no new predictions over Coulomb's theory in static situations, that is when charges are not moving or accelerating. The equation

$$E = k\frac{|Q|}{r^2}$$

only holds in static situations.
- When charges are moving or accelerating, the above equation no longer holds. If charged particles move slowly, it's a good approximation. As charges approach the speed of light, it fails completely, and the electric field must be found by some other means.
- The reason I sometimes call the above equation Coulomb's law is that it makes the same predictions as the original Coulumb's law.

Purpose of the electric field

- If a charged particle wiggles in one place, that modifies the electric field produced by the particle. Changes ripple through the electric field at the speed of light, and only later affect a second particle. In this course, We will not study the equations that describe this behavior.
- Faraday and Maxwell made electromagnetic theory into a field theory that removed the need for action at a distance, that explained the relationships between electricity and magnetism, that predicted radiation, and that gave a theory of light. The electric field is now viewed as a small price price to pay for all of these benefits.