Circuits

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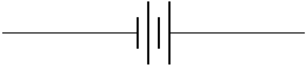
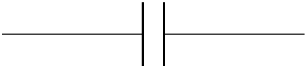
A circuit is made by connecting

wires	
batteries	
resistors	
capacitors	

Two key ideas in circuit analysis

- Voltage = difference in electric potential (Volts)
- Current = charge per unit time (Amps)

Electric current is the rate at which charge flows.

- Analogy: electric current is like water flowing through a pipe.
  - The rate at which water flows could be measured in gallons per minute.
  - The rate at which current flows is measured in Coulombs per second (Amps).
- Unit of electric current: Ampere (Amp, A)
  - $1 \text{ A} = 1 \text{ C/s}$
- Symbol for electric current: I
- Current doesn't get used up.

Wires

- Wires are made from good electrical conductors, usually metals
- A perfect conductor has no electric field inside it, so it has a single value of electric potential.
- When analyzing a circuit, we label each wire with a known or unknown electric potential.
- We can pick one wire to be zero electric potential.
- An ideal wire (perfect conductor) can carry any amount of current.

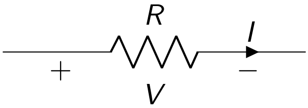
Resistors

- A resistor obeys Ohm's law.

$$V = IR$$

- V is the voltage across the resistor, meaning the difference in electric potential between the two ends.
- I is the current flowing through the resistor.
- R is the resistance of the resistor (Ohms)

A resistor satisfies Ohm's law.



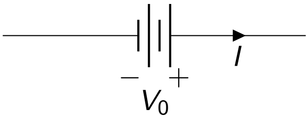
$$V = IR$$

- V is the voltage across the resistor. The $+$ and $-$ mean that if $V > 0$ then the electric potential is higher on the left. If $V < 0$, then the electric potential is higher on the right.
- I is the current through the resistor. If $I > 0$, then (positive) current flows to the right. If $I < 0$, then current flows to the left.
- If the arrow for I is to the right, then the $+$ for V must be on the left. Otherwise $V = IR$ fails and needs a minus sign, which we don't want.

Batteries

- A battery supplies a fixed amount of voltage.
- An ideal battery can supply any amount of current (positive or negative).

A battery is a constant voltage source.



- We assume $V_0 > 0$. Electric potential is higher at the long bar.
- If $I > 0$, the battery is supplying power to the rest of the circuit. If $I < 0$, the battery is being recharged.

Resistivity

- is an intrinsic property of a material.

Material	Resistivity, ρ
Silver	1.59×10^{-8} Ω·m
Copper	1.68×10^{-8} Ω·m
Gold	2.44×10^{-8} Ω·m
Tungsten	5.6×10^{-8} Ω·m

- For a cylinder of material,

$$R = \frac{\rho L}{A}$$

  - L = length of cylinder
  - A = cross-sectional area of cylinder

Power

- The power supplied by a battery is

$$P = IV$$

  where V is the voltage of the battery and I is the current through the battery.
- The power dissipated by a resistor is

$$P = IV = I^2R = \frac{V^2}{R}$$

  where V is the voltage across the resistor and I is the current through the resistor.

Goals for analyzing circuits

- Find the voltage across every resistor.
- Find the current through every resistor.
- Find the current through every battery.
- We also want to know the voltage across every battery, but we are usually given that information.

Strategy for analyzing circuits the hard way

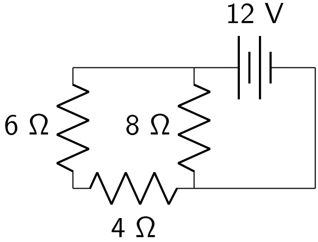
1. Label the electric potential of every wire. Use a numerical value if possible, otherwise a variable such as V_A or V_B. You may choose one wire to have zero electric potential. I like to choose the low-potential end of a battery.
   - If you can label the electric potential of every wire with a number of volts (and not just a variable), you are almost done.
2. Use Ohm's law to find the current through every resistor, either as a number or as an expression with variables.
3. Use Kirchhoff's current law (current flowing into a node equals current flowing out) to write down equations.
4. Solve for the variables.

Series and Parallel Circuit Elements

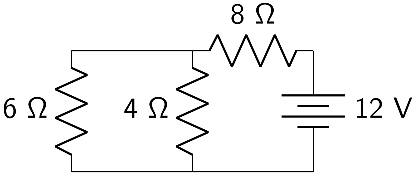
- A new tool that can make analysis of circuits easier.
- Two circuit elements are in series if one end of one is connected to one end of the other with nothing else connected there.
- Elements in series carry the same current, because there is no where else for the current to flow.
- Two circuit elements are in parallel if their left ends are connected and their right ends are also connected. Anything else may also be connected at either end.
- Elements in parallel have the same voltage, because they have the same difference in electric potential.

Circuit Elements in Series

- Two circuit elements are in series if one end of one is connected to one end of the other with nothing else connected there.
- Elements in series carry the same current, because there is no where else for the current to flow.



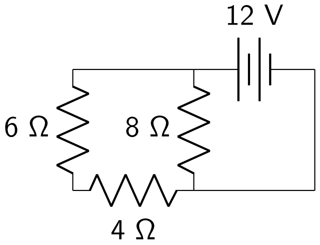
- 4Ω and 6Ω are in series



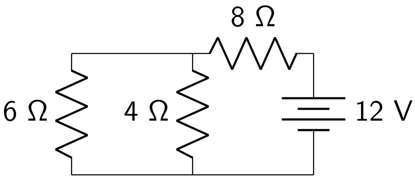
- 12 V battery and 8Ω resistor are in series

Circuit Elements in Parallel

- Two circuit elements are in parallel if their left ends are connected and their right ends are also connected. Anything else may also be connected at either end.
- Elements in parallel have the same voltage, because they have the same difference in electric potential.



- 12 V battery and 8Ω resistor are in parallel



- 4Ω and 6Ω are in parallel

Resistors in Series and Parallel

- Two resistors in series are equivalent to a single resistor with resistance
$$R_{\mathrm{eq}} = R_1 + R_2.$$
- Two resistors in parallel are equivalent to a single resistor with resistance
$$R_{\mathrm{eq}} = \frac{R_1 R_2}{R_1 + R_2}.$$

Revised strategy for analyzing circuits that may be easier

1. Label the electric potential of every wire. Use a numerical value if possible, otherwise a variable. Choose one wire to have zero electric potential.
2. If you can label the electric potential of every wire with a number of volts (and not just a variable), do it.
   - Find all the voltages by subtracting the electric potentials.
   - Find the currents through the resistors by using Ohm's law.
   - Find the currents through the batteries by using Kirchhoff's current law.
   - You've succeeded in analyzing the circuit.

Revised strategy, continued

3. If there is a variable somewhere for electric potential, look for a series or parallel combination of resistors. If you find such a combination, make a simpler equivalent circuit. Use this same strategy on the simplified circuit (start by labeling electric potentials) to find all the potentials and currents you can. If there are still variables for electric potential, try to find another series or parallel combination and simplify again. Once you find all the potentials and currents on the simplified circuit, bring them back up to the next most complex circuit, use Ohm's law and Kirchhoff's current law to find all the potentials and currents for that circuit. Eventually, you will arrive back at the original circuit and can find all the potentials and currents.