Simple Harmonic Motion

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October 29, 2024

Springs

- A spring has an equilibrium length where it's not being stretched or compressed.
- You can extend a spring by pulling on one end, with the other end fixed to a wall, let's say.
- By exerting a certain force on the spring, the spring stretches a certain amount.
- If you want to get the spring to stretch more, you need to apply a larger force.
- There is not a single force associated with a spring. You can pull on it with a small force and get a small displacement, or you can pull on it with a big force and get a big displacement.

Hooke's Law

- Most springs have a range over which the displacement (the stretching or compressing) is a linear function of the applied force. In other words, if you double the force, you get double the displacement.
- Springs operating in this linear regime are said to be *Hooke's law* springs, or to be obeying *Hooke's law*.
- Here is Hooke's law:

$$F = kx$$

- F is the force pulling on the spring.
- x is the displacement of the spring from equilibrium (measured in meters).
- k is called the *spring constant*, and is a property of the spring. Springs with a large k are very stiff springs. Springs with a small k are easy to extend.

A spring can oscillate.

A mass m on the end of a (massless) spring with spring constant k will oscillate with period

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

If the spring itself has a mass m_s, then the equation for the period gets modified as follows.

$$T = 2\pi\sqrt{\frac{m + \frac{1}{3}m_s}{k}}$$

Finding the spring constant k: Method 1

Start with Hooke's law.

$$F = kx$$

Suppose we use different masses m which produce different forces F. We don't let the spring oscillate. We just want the mass to hang, limply, on the end of the spring. We can measure the displacement x for each mass m and make a plot of F on the vertical axis versus x on the horizontal axis. The slope of this graph is k, so we can find the spring constant this way.

Finding the spring constant k: Method 2

Start with this equation from the oscillation slide.

$$T = 2\pi\sqrt{\frac{m + \frac{1}{3}m_s}{k}}$$

Suppose we use different masses m and measure their periods T. With this data, we'll make a plot of T^2 on the vertical axis versus m on the horizontal axis.

$$T^2 = \frac{4\pi^2}{k}m + \frac{4\pi^2 m_s}{3k}$$

The slope of this graph is $4\pi^2/k$ and the y-intercept is $4\pi^2 m_s/(3k)$. If you find the slope, you can figure out k.