Ballistic Pendulum

Scott N. Walck

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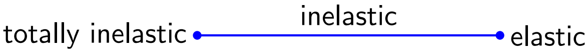
History of the Ballistic Pendulum

- Invented in 1742
- Published in the book *New Principles of Gunnery*, by English mathematician Benjamin Robins
- First way to accurately measure the velocity of a bullet
- Physicists today like the ballistic pendulum because it's a beautiful example of conservation of energy, conservation of momentum, and how they are different.
- Basic idea: fire a bullet into a block of wood that is hanging from a rope, and see how high it swings.
- We will learn later that collisions are a great opportunity to apply conservation of momentum.

Types of Collisions

- Totally inelastic collision: objects stick together
- Inelastic collision
- Elastic collision: kinetic energy is conserved
  - An elastic collision is *not* simply one in which things bounce off of each other.



Kinetic Energy

An object with mass m and speed v has kinetic energy

$$\mathrm{KE} = \frac{1}{2} m v^2.$$

A system of two objects has kinetic energy

$$\mathrm{KE} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2.$$

Equation 1

$$mv = (M + m)V$$

Situation: a collision. One object has mass m and is moving with one-dimensional velocity v toward a second object. The second object has mass M and is initially at rest. After the collision, the two objects stick together and move together with one-dimensional velocity V.

m	mass of the incoming object
M	mass of the initially stationary object
v	initial 1D velocity of the incoming object
V	final 1D velocity after the collision

- This equation is an example of a "conservation of momentum" equation.

Equation 2

$$\frac{1}{2}(M+m)V^2 = (M+m)gh$$

Situation: Two objects, with masses M and m are stuck together and moving together. They are moving upward and slowing down. At the end, they have no velocity, but they are higher than at the start.

m	mass of the first object
M	mass of the second object
g	acceleration of gravity (9.8 $\mathrm{m/s^2}$)
V	initial 1D velocity of the two objects
h	increase in height of the objects

- This equation is an example of a "conservation of energy" equation.

Equation 3

$$V = \sqrt{2gh}$$

Situation: An object (or two objects stuck together) is moving upward and slowing down. At the end, it has no velocity, but is higher than at the start.

V	initial 1D velocity of the object
g	acceleration of gravity (9.8 m/s^2)
h	increase in height of the object

- You can get this equation from equation 2 using algebra.

Equation 4

$$v = \frac{M + m}{m}\sqrt{2gh}$$

Situation: a collision followed by swinging. One object has mass m and is moving with one-dimensional velocity v toward a second object. The second object has mass M and is initially at rest. After the collision, the two objects stick together and move together as they swing upward, moving vertically a distance h to reach their highest point.

m	mass of the incoming object
M	mass of the initially stationary object
v	initial 1D velocity of the incoming object
g	acceleration of gravity (9.8 $\mathrm{m/s^2}$)
h	increase in height of the object

- You can get this equation from equation 1 and equation 3 using algebra.

Equation 5

$$R = vt$$

Situation: projectile motion.

R	horizontal range (displacement) of the projectile
v	horizontal component of the velocity (constant)
t	time from launch to landing

- You can get this equation from the position-time equation in the x direction,

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2,$$

by realizing that $a_x = 0$ for projectile motion, choosing $x_0 = 0$, using the symbol v for v_{x0}, and using the symbol R for x.

Equation 6

$$H = \frac{1}{2}gt^2$$

Situation: projectile motion in which the initial velocity is horizontal.

H	height from which projectile is launched
g	acceleration of gravity (9.8 $\mathrm{m/s^2}$)
t	time from launch to landing

- You can get this from

$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

  by choosing a coordinate system in which the positive y direction is downward, substituting $a_y = g$, realizing that $v_{y0} = 0$ because the projectile was launched horizontally, choosing $y_0 = 0$, and using the symbol H for y.

Equation 7

$$v = \frac{R}{\sqrt{\frac{2H}{g}}}$$

Situation: projectile motion in which the initial velocity is horizontal.

v	initial speed of the projectile
R	horizontal range (displacement) of the projectile
H	height from which projectile is launched
g	acceleration of gravity (9.8 m/s^2)

- You can get this equation by solving equation 6 for t, plugging the result into equation 5, and doing a little more algebra.

Equation 8

$$h = h_2 - h_1$$

Situation: We want the height that h_2 is above h_1. For this lab, the heights refer to the pendulum.

h_1	height of indicator when pendulum is hanging vertically
h_2	height of indicator when pendulum is highest
h	change in vertical displacement of pendulum

- You can get this equation by using common sense.