Universal Gravity

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Four theories of gravity

1. Gravity accelerates objects downward at 9.8 $\mathrm{m/s^2}$
2. Gravity is a force near Earth's surface, $F_{\mathrm{G}} = mg$
3. Gravity is a force between any two objects, $F_{\mathrm{G}} = G\frac{m_1 m_2}{r^2}$
4. Gravity is the curvature of spacetime (Einstein, 1915)

List of forces so far

Force	symbol	direction	magnitude
somebody pushing	F_{P}		
gravity (weight)	F_{G}	downward	$F_{\mathrm{G}} = mg$
universal gravity	F_{G}	toward other object	$F_{\mathrm{G}} = G\frac{m_1 m_2}{r^2}$
normal force	F_{N}	$\perp$ to surface	
tension	F_{T}	along rope	
air resistance		opposite the velocity	
kinetic friction	F_{fr}	opposite the velocity	$F_{\mathrm{fr}} = \mu_{\mathrm{k}} F_{\mathrm{N}}$
static friction	F_{fr}	$\parallel$ to surface	$F_{\mathrm{fr}} \leq \mu_{\mathrm{s}} F_{\mathrm{N}}$

How is G related to g?

$$g = \frac{GM_{\mathrm{E}}}{R_{\mathrm{E}}^2}$$

Exercise: Calculate the acceleration of gravity on the Moon, g_{Moon}.

Satellite motion

Bring together 3 ideas:

- Newton's second law, $F = ma$
- Universal gravity, $F_{\mathrm{G}} = G\frac{m_1 m_2}{r^2}$
- Uniform circular motion, $a = \frac{v^2}{r}$

For a satellite of mass m attracted by another object with mass M,

$$G\frac{Mm}{r^2} = m\frac{v^2}{r}$$

- On the left, r is the distance between the centers of the two objects.
- On the right, r is the radius of circular motion.
- If the satellite is small, these two rs are the same.

Small satellite

$$G\frac{Mm}{r^2} = m\frac{v^2}{r}$$

Relation between satellite speed and radius:

$$GM = v^2 r$$

Using $v = 2\pi r/T$, get a relation between radius and period:

$$GMT^2 = 4\pi^2 r^3$$

Exercises

1. Calculate the period of a satellite in low Earth orbit.
2. Calculate the period of the Moon.