Rotational Motion

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Rotational Quantities

Linear Qty	Symbol	Unit	Rotational Quantity	Symbol	Unit
Position	x	m	Angular Position	θ	rad
Velocity	v	m/s	Angular Velocity	ω	rad/s
Acceleration	a	m/s^2	Angular Acceleration	α	rad/s^2
Mass	m	kg	Moment of Inertia	I	kg m^2
Force	F	N	Torque	τ	N m
Net Force	$F_{net} = ma$	N	Net Torque	$\tau_{net} = I\alpha$	N m
Trans. KE	$\text{KE} = \frac{1}{2}mv^2$	J	Rotational KE	$\text{KE} = \frac{1}{2}I\omega^2$	J
Momentum	$p = mv$	kg m/s	Angular Momentum	$L = I\omega$	J s

Rotational Kinematics

Linear Qty		Unit	Rotational Quantity		Unit
Position	x	m	Angular Position	θ	rad
Velocity	v	m/s	Angular Velocity	ω	rad/s
Acceleration	a	m/s^2	Angular Acceleration	α	rad/s^2

Degrees and radians both measure angle.

A radian is the angle that a length of one radius makes on the circumference of a circle.

$$2\pi \text{ radians} = 1 \text{ circle} = 360^\circ$$

In mathematics and physics, radians are often a superior unit for angles.

If angular acceleration is constant, use the CAA equations.

- Velocity-Time Equation

$$\omega = \omega_0 + \alpha t$$

- Position-Time Equation

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

- Position-Velocity Equation

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Meaning of symbols in CAA equations

t	the time	independent variable
θ	angular position at time t	dependent variable
ω	angular velocity at time t	dependent variable
α	the constant angular acceleration	parameter
θ_0	angular position at time 0	parameter
ω_0	angular velocity at time 0	parameter

Rotation about a fixed axis

$$\ell = r\theta$$
$$v = r\omega$$
$$a_{\mathsf{tan}} = r\alpha$$

- r is the distance from the axis to a point we care about
- θ is an angle through which the point has rotated
- ω is the angular velocity of the rotation
- α is the angular acceleration of the rotation
- ℓ is the distance traveled for the point we care about
- v is the speed of the point we care about
- a_{tan} is the tangential component of (linear) acceleration for the point we care about

If I had my way

I would define a constant θ_R equal to one radian.

$$\theta_R = 1 \text{ rad} = \frac{1}{2\pi} \text{ rev} = \frac{360^\circ}{2\pi}$$

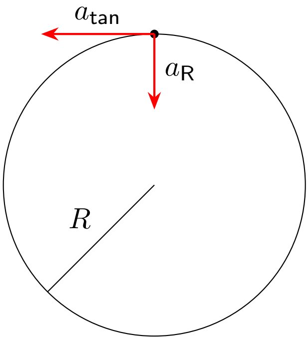
I would make the equations slightly more complicated.

$$\begin{aligned} \ell &= r\frac{\theta}{\theta_R} \\ v &= \frac{r\omega}{\theta_R} \\ a_{\mathsf{tan}} &= \frac{r\alpha}{\theta_R} \end{aligned}$$

The benefit of these equations is that you can use whatever angle units you want (degrees, radians, or revolutions).

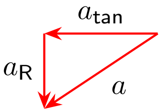
Nonuniform Circular Motion

Rotation about a fixed axis



$$a_{\mathsf{tan}} = R\alpha$$

$$a_{\mathsf{R}} = \frac{v^2}{R} = \omega^2 R$$



- a is the linear acceleration of a point that is R from the axis.

Two meanings for $v = r\omega$

1. Fixed axis of rotation
   - r is the distance from the axis to a point we care about
   - ω is the angular velocity of the rotation
   - v is the speed of the point we care about
2. Rolling motion
   - r is the radius of the wheel
   - ω is the angular velocity of the rotation
   - v is the speed of the center of the wheel

Torque is the strength of a twist.

- A torque is clockwise or counterclockwise.
- Symbol for torque: τ
- SI unit for torque: N m (not J)

A force can produce a torque.

$$\tau = rF\sin\theta$$

- F is the magnitude of the force.
- r is the distance from where the force acts to the axis of rotation.
- θ is the angle between r and F: $0^\circ \leq \theta \leq 180^\circ$
- τ is clockwise or counterclockwise (about the axis of rotation)

Moment of Inertia

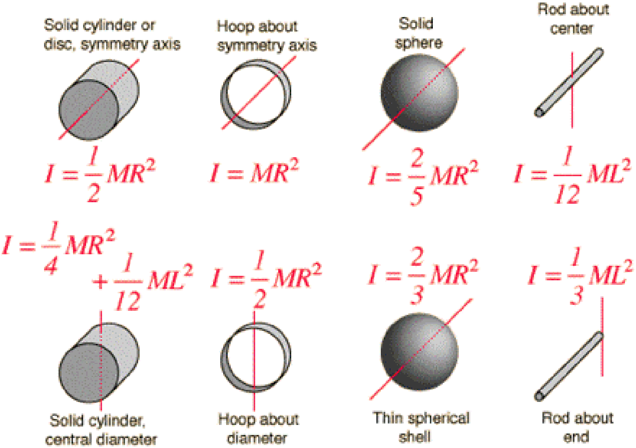
- Just like an object has a mass, it has a *moment of inertia*.
- Just as an object's *mass* quantifies the difficulty of producing *acceleration* of the object, an object's *moment of inertia* quantifies the difficulty of producing *angular acceleration* of the object.
- An object with a large *mass* requires a large *force* to experience *acceleration*. An object with a large *moment of inertia* requires a large *torque* to experience *angular acceleration*.
- Symbol for moment of inertia: I
- SI unit for moment of inertia: kg m^2
- Don't confuse *inertia* with *moment of inertia*.

Moment of Inertia depends on axis of rotation.

$$I = \sum mr^2 = m_1 r_1^2 + m_2 r_2^2 + \cdots$$

- r_1 is distance from m_1 to axis of rotation
- r_2 is distance from m_2 to axis of rotation

Moments of inertia for common objects



Rotational version of Newton's 2nd law

$$\tau_{\text{net}} = I\alpha$$

Two types of kinetic energy

- Translational kinetic energy

$$\mathrm{KE}_{\mathrm{trans}} = \frac{1}{2} m v^2$$

- Rotational kinetic energy

$$\mathrm{KE}_{\mathrm{rot}} = \frac{1}{2} I \omega^2$$

- Total kinetic energy is the sum of the two

$$\mathrm{KE} = \mathrm{KE}_{\mathrm{trans}} + \mathrm{KE}_{\mathrm{rot}}$$

Angular Momentum

- Symbol: L
- SI Unit: kg m^2/s or J s

$$L = I\omega$$

- Ice skater example