Momentum

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Momentum is mass times velocity

The momentum of an object is the product of the object's mass and the object's velocity.

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

- Momentum is a vector.
- SI units: kg m/s
- Relativity (Einstein, 1905) uses a different definition.

Momentum of a system of objects

The momentum of a system of two objects is the vector sum of the momenta of each object.

$$\vec{\mathbf{P}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$$

Newton's Second Law, Momentum Version

Net force equals change in momentum per unit time.

$$\vec{\mathbf{F}}_{\text{net}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$$

The change in momentum of an object is the net force on the object times the time over which the net force acts.

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}_{\text{net}}\Delta t$$

Internal and External Forces

A system of objects is just a choice of which objects to pay attention to.

- An internal force is a force that acts *on* an object in the system and is produced *by* an object in the system.
- An external force is a force produced by something outside the system.

Criterion for Momentum Conservation

If no external forces are present, system momentum is conserved.

- Collisions: If two objects collide, and we choose our system to include both objects, then the force of collision is an internal force, so system momentum is conserved.

System Momentum is Conserved in a Collision

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

Why is Momentum Conserved in a Collision?

- Collisions are short; they happen quickly.
- The force of one object colliding with another is an *internal* force if you take the system to consist of both objects.
- Our criterion for momentum conservation says that system momentum is conserved if no external forces act on objects in the system. But even if there are external forces, such as gravity or a normal force, in a collision they usually act for such a short time that they have a negligible effect on the system momentum.

Don't confuse conservation of momentum with conservation of mechanical energy.

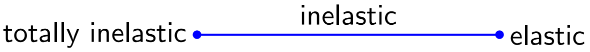
- The criteria for mechanical energy conservation have to do with whether forces are *conservative* or *nonconservative* (and then whether the nonconservative forces do work).
- The criterion for momentum conservation has to do with whether forces are *internal* or *external*. Momentum conservation doesn't care whether they are conservative or nonconservative.
- The words *conservative* and *nonconservative*, as applied to forces, only concern energy, not momentum.

Collision in One Dimension

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Types of Collisions

- Totally inelastic collision: objects stick together
- Inelastic collision
- Elastic collision: kinetic energy is conserved
  - An elastic collision is *not* simply one in which things bounce off of each other.



Totally Inelastic Collision

Start with general collision in 1D:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Objects stick together after collision, so

$$v_{1f} = v_{2f} = v_f$$

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

Elastic Collision: Relative Velocity Flips

- An elastic collision is one in which kinetic energy is conserved.

General collision in 1D:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Relative velocity flips in an elastic collision:

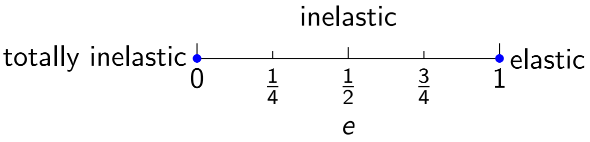
$$v_{1f} - v_{2f} = -(v_{1i} - v_{2i})$$

Solve these two equations to find the two unknowns, v_{1f} and v_{2f}.

Coefficient of Restitution

$$e = -\frac{v_{1f} - v_{2f}}{v_{1i} - v_{2i}}$$

- Totally inelastic collision: objects stick together ($e = 0$)
- Inelastic collision ($0 < e < 1$)
- Elastic collision: kinetic energy is conserved ($e = 1$)



Center of Mass

Center of mass is a weighted average of the positions of two or more objects, where the weighting is done by mass.

- Center of mass of two things:

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \qquad y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

- Center of mass of three things:

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \quad y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$