Mechanical Energy

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Types of Energy

Kinetic energy	ordered
Potential energy	ordered
Thermal energy	disordered
Electromagnetic energy	ordered
Chemical energy	ordered
Nuclear energy	ordered
Mass energy	ordered

Energy Units

The SI unit for energy is the Joule (J).

kWh	3,600,000 J
Cal	4190 J
Btu	1056 J
mWh	3.6 J
eV	1.6022×10^{-19} J

Scales of Energy

80 kJ	laptop battery
10 MJ	daily human food intake (2400 Cal)
200 MJ	average daily human energy use (world)
1000 MJ	average daily human energy use (US)
500 EJ	yearly global energy use

Laptop battery: 24000 mWh $\approx$ 80 kJ

Kinetic Energy

An object with mass m and speed v has kinetic energy

$$\mathrm{KE} = \frac{1}{2} m v^2.$$

A system of two objects has kinetic energy

$$\mathrm{KE} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2.$$

Kinetic energy belongs to an object.

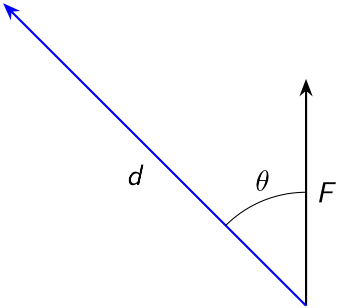
Properties of an object:

- mass
- position
- velocity
- kinetic energy

Work

A force can do work on an object. Work, like energy, is measured in Joules (J). The work done by a force F on an object experiencing a displacement d is

$$W = Fd\cos\theta.$$



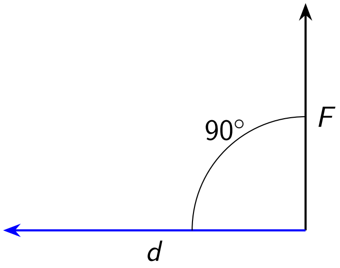
No Displacement, No Work

If an object doesn't move, forces may be acting on it, but none of them do any work.

$$W = Fd\cos\theta$$

If $d = 0$, then $W = 0$.

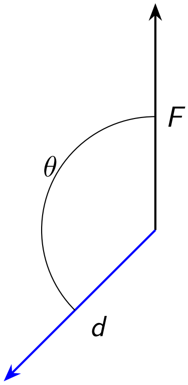
A force perpendicular to displacement does no work.



$$W = Fd\cos\theta$$

If $\theta = 90^\circ$, then $W = 0$.

A force can do negative work.



$$W = Fd\cos\theta$$

If $\theta > 90^\circ$, then $W < 0$.

Net Work

- The *net work* done on an object is the sum of the work done by each force that acts on the object.
- Good news: Work and energy are scalars, so we are just adding (positive or negative) numbers here.

Analogy

Each object has a checking account that contains its kinetic energy.
The checking account can go to zero, but it cannot go negative.

Physics	Banking
kinetic energy	checking account balance
work done by gravity	deposit or withdrawal by gravity
work done by Amy pushing	deposit or withdrawal by Amy
work done by tension	deposit or withdrawal by tension
work done by normal force	deposit or withdrawal by normal force

Work-KE Theorem

The net work done on an object is equal to the change in its kinetic energy.

$$W_{\text{net}} = \Delta\text{KE}$$

Applied to one object:

$$W_{\text{net}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Conservative and nonconservative forces

Force	Conservative?	Potential Energy
Gravity	Conservative	$\mathrm{PE} = mgy$
Normal Force	Nonconservative	none
Rope Tension	Nonconservative	none
Friction	Nonconservative	none
Spring	Conservative	$\mathrm{PE} = \frac{1}{2}kx^2$

Mechanical Energy

The mechanical energy of a system is the sum of its kinetic energy and its potential energy.

$$\mathrm{ME} = \mathrm{KE} + \mathrm{PE}$$

For one object,

$$\mathrm{ME} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2.$$

The $\frac{1}{2}kx^2$ is only included when a spring is present.

Mechanical Energy for a System of Objects and Springs

Include in mechanical energy:

- $\frac{1}{2}mv^2$ for each object
- mgy for each object (assuming all objects are near Earth's surface)
- $\frac{1}{2}kx^2$ for each spring

For two objects and one spring,

$$\mathrm{ME} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + mgy_1 + mgy_2 + \frac{1}{2}kx^2$$

Work-ME Theorem

The net work done on an object by nonconservative forces is equal to the change in its mechanical energy.

$$W_{\text{net,NC}} = \Delta\text{ME}$$

Applied to one object with one spring:

$$W_{\text{net,NC}} = \frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2 - \frac{1}{2}mv_i^2 - mgy_i - \frac{1}{2}kx_i^2$$

Mechanical energy is conserved if

1. no nonconservative forces are present, or
2. nonconservative forces are present but they do no work, or
3. nonconservative forces are present and do work but the work done by some exactly cancels the work done by others.

Conservation of Mechanical Energy

If mechanical energy is conserved:

$$\mathrm{KE}_i + \mathrm{PE}_i = \mathrm{KE}_f + \mathrm{PE}_f$$

If mechanical energy is conserved for one mass and one spring, the previous equation expands into this:

$$\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2$$

Analogy between Mechanical Energy and Banking

Each physical system has a checking account and a savings account. The checking account can go to zero, but it cannot go negative. The savings account can go negative if you borrow money.

Physics	Banking
kinetic energy	checking account balance
potential energy	savings account balance
work done by gravity	not a transaction
work done by Amy pushing	deposit or withdrawal by Amy
work done by tension	deposit or withdrawal by tension
work done by normal force	deposit or withdrawal by normal force

Using Mechanical Energy as a Tool

- Check if mechanical energy is conserved using the 3 criteria.
  - If mechanical energy is conserved:

$$\mathrm{KE}_i + \mathrm{PE}_i = \mathrm{KE}_f + \mathrm{PE}_f$$

  - If mechanical energy is not conserved:

$$\begin{aligned} W_{\text{net,NC}} &= \Delta \mathrm{ME} \\ &= \mathrm{KE}_f + \mathrm{PE}_f - \mathrm{KE}_i - \mathrm{PE}_i \end{aligned}$$

Power

- *Power* is energy transformed per unit time.
- *Power* can be work per unit time.
- The power imparted by a force F acting on an object with velocity v is
$$P = Fv\cos\theta,$$
where θ is the angle between the force and the velocity.

Power vs. Energy

Wrong question

- How much power does the nuclear plant produce in a day?
- How much energy can the plant continuously produce?

Right question

- How much energy does the nuclear plant produce in a day?
- How much power can the plant continuously produce?