# Equation Sheet

## Quantum Mysteries

$$f = \frac{1}{T}$$

$$v = \frac{\lambda}{T} = \lambda f$$

$$E = hf$$

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.626 \times 10^{-34} \text{ J s} = 4.136 \times 10^{-15} \text{ eV s}$$

Speed of sound in air: 343 m/s

$$i^2 = -1$$

If $z = x + iy$, then:

$$z^* = x - iy$$

$$z^{-1} = \frac{1}{z} = \frac{x - iy}{x^2 + y^2}$$

$$|z| = \sqrt{x^2 + y^2}$$

$$\sqrt{z} = \begin{cases} \sqrt{\frac{\sqrt{x^2+y^2}}{2}} \left( \sqrt{1 + \frac{x}{\sqrt{x^2+y^2}}} + i\sqrt{1 - \frac{x}{\sqrt{x^2+y^2}}} \right) & \text{for} \quad y \geq 0 \\ \sqrt{\frac{\sqrt{x^2+y^2}}{2}} \left( -\sqrt{1 + \frac{x}{\sqrt{x^2+y^2}}} + i\sqrt{1 - \frac{x}{\sqrt{x^2+y^2}}} \right) & \text{for} \quad y \leq 0 \end{cases}$$

$$\Sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \Sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \Sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

| | $1$ | $\Sigma_1$ | $\Sigma_2$ | $\Sigma_3$ |
|---|---|---|---|---|
| $1$ | $1$ | $\Sigma_1$ | $\Sigma_2$ | $\Sigma_3$ |
| $\Sigma_1$ | $\Sigma_1$ | $1$ | $\Sigma_1\Sigma_2 = i\Sigma_3$ | $-i\Sigma_2$ |
| $\Sigma_2$ | $\Sigma_2$ | $-i\Sigma_3$ | $1$ | $i\Sigma_1$ |
| $\Sigma_3$ | $\Sigma_3$ | $i\Sigma_2$ | $-i\Sigma_1$ | $1$ |

If

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = z_0 1 + z_1\Sigma_1 + z_2\Sigma_2 + z_3\Sigma_3$$

then

$$\begin{aligned} a_{11} &= z_0 + z_3 & a_{21} &= z_1 + iz_2 \\ a_{22} &= z_0 - z_3 & a_{12} &= z_1 - iz_2 \\ z_0 &= \frac{1}{2}(a_{11} + a_{22}) & z_1 &= \frac{1}{2}(a_{12} + a_{21}) \\ z_3 &= \frac{1}{2}(a_{11} - a_{22}) & z_2 &= \frac{i}{2}(a_{12} - a_{21}). \end{aligned}$$

$$(x_1\Sigma_1 + x_2\Sigma_2 + x_3\Sigma_3)^2 = (x_1^2 + x_2^2 + x_3^2)1$$

If

$$M = z_0 1 + z_1\Sigma_1 + z_2\Sigma_2 + z_3\Sigma_3$$

then

$$M^{-1} = \frac{z_0 1 - z_1\Sigma_1 - z_2\Sigma_2 - z_3\Sigma_3}{z_0^2 - z_1^2 - z_2^2 - z_3^2}.$$

$$\langle x \rangle = \sum_x x\rho(x) \qquad \langle x^2 \rangle = \sum_x x^2\rho(x)$$

$$\Big\langle (x - \langle x \rangle)^2 \Big\rangle = \langle x^2 \rangle - \langle x \rangle^2$$

The possible values of $M = x_0 1 + x_1 \Sigma_1 + x_2 \Sigma_2 + x_3 \Sigma_3$ are

$$x_0 - r, x_0 + r$$

where

$$r = \sqrt{x_1^2 + x_2^2 + x_3^2}.$$

The probabilities for these outcomes are

$$\rho(x_0 - r) = \frac{1}{2} - \frac{1}{2}\langle U \rangle \qquad \rho(x_0 + r) = \frac{1}{2} + \frac{1}{2}\langle U \rangle$$

where

$$U = \frac{x_1}{r}\Sigma_1 + \frac{x_2}{r}\Sigma_2 + \frac{x_3}{r}\Sigma_3.$$

If $M$ has the definite value $x_0 - r$, then

$$\begin{aligned}
\langle \Sigma_1 \rangle &= -\frac{x_1}{r} \\
\langle \Sigma_2 \rangle &= -\frac{x_2}{r} \\
\langle \Sigma_3 \rangle &= -\frac{x_3}{r}.
\end{aligned}$$

If $M$ has the definite value $x_0 + r$, then

$$\begin{aligned}
\langle \Sigma_1 \rangle &= \frac{x_1}{r} \\
\langle \Sigma_2 \rangle &= \frac{x_2}{r} \\
\langle \Sigma_3 \rangle &= \frac{x_3}{r}.
\end{aligned}$$