Wave-Particle Duality and an Introduction to Quantum Mechanics

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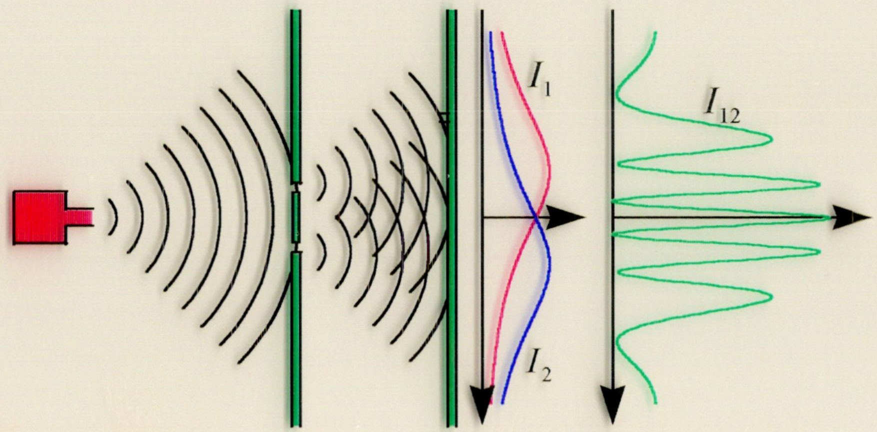
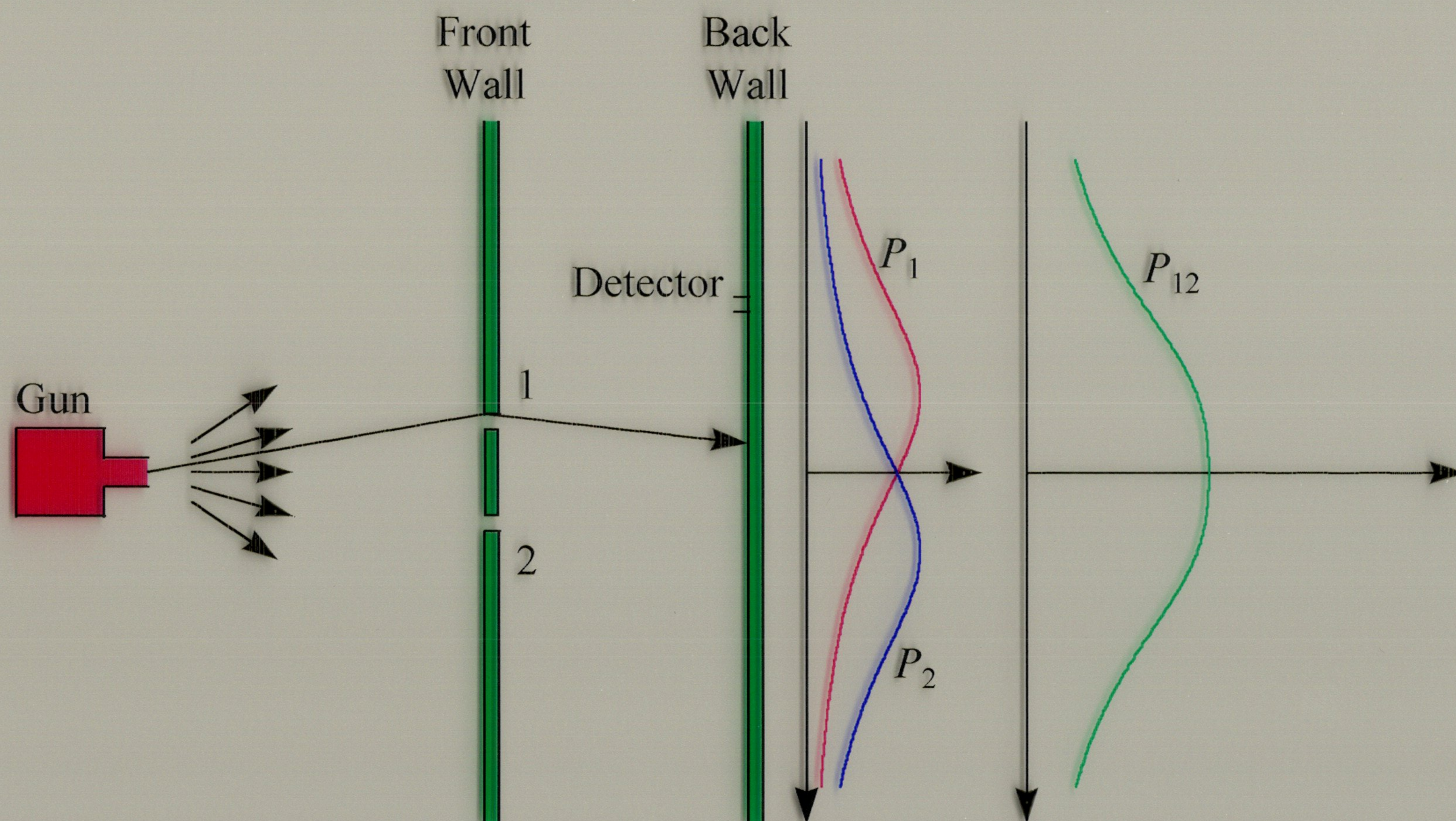
Thanks to
Richard Feynman,
Feynman Lectures

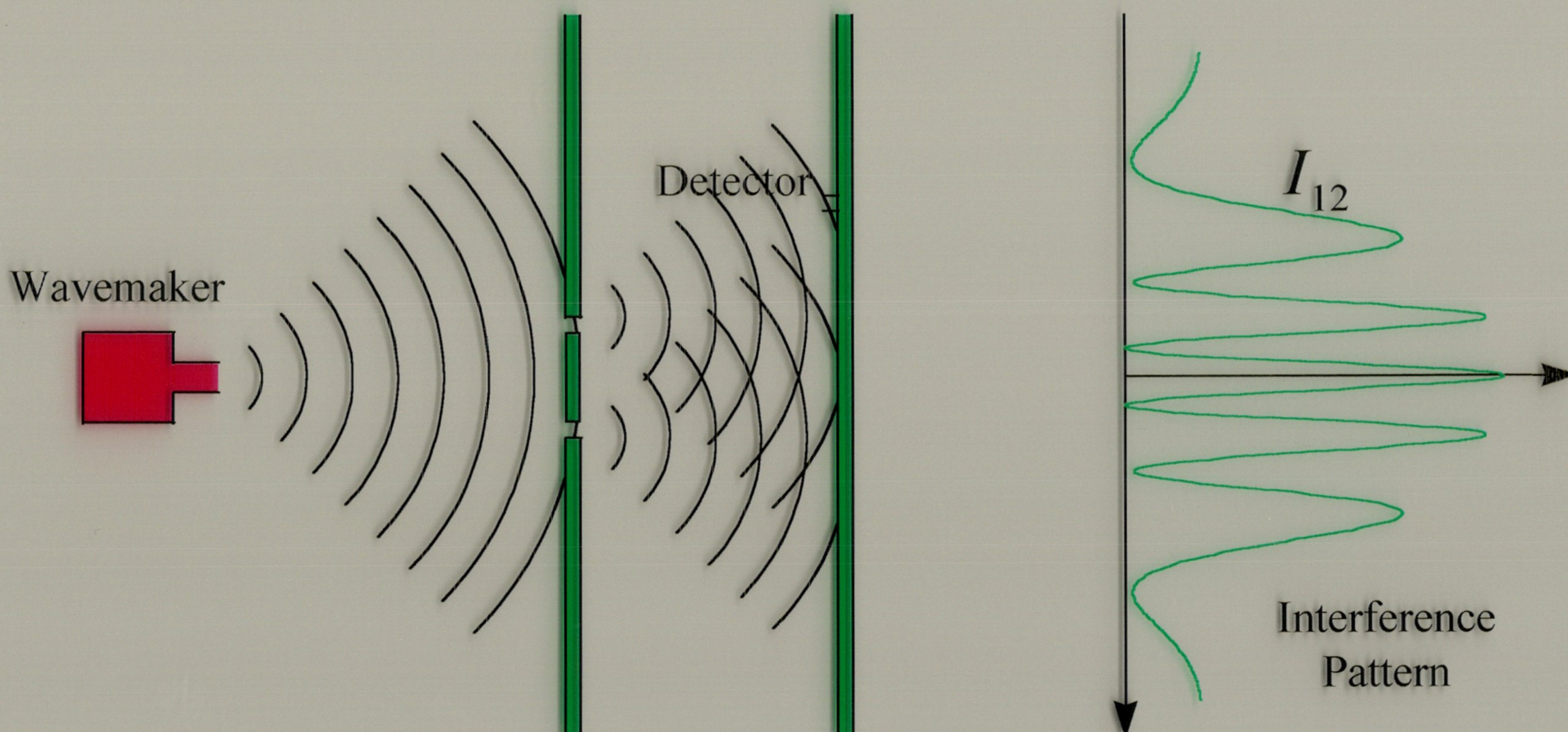


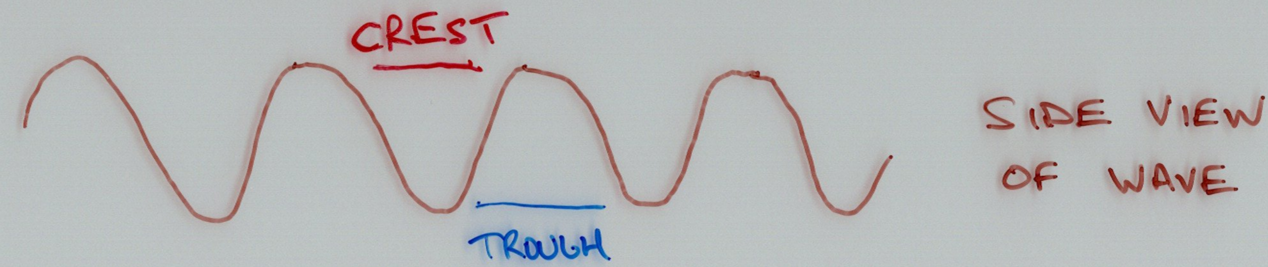
An Experiment with Particles

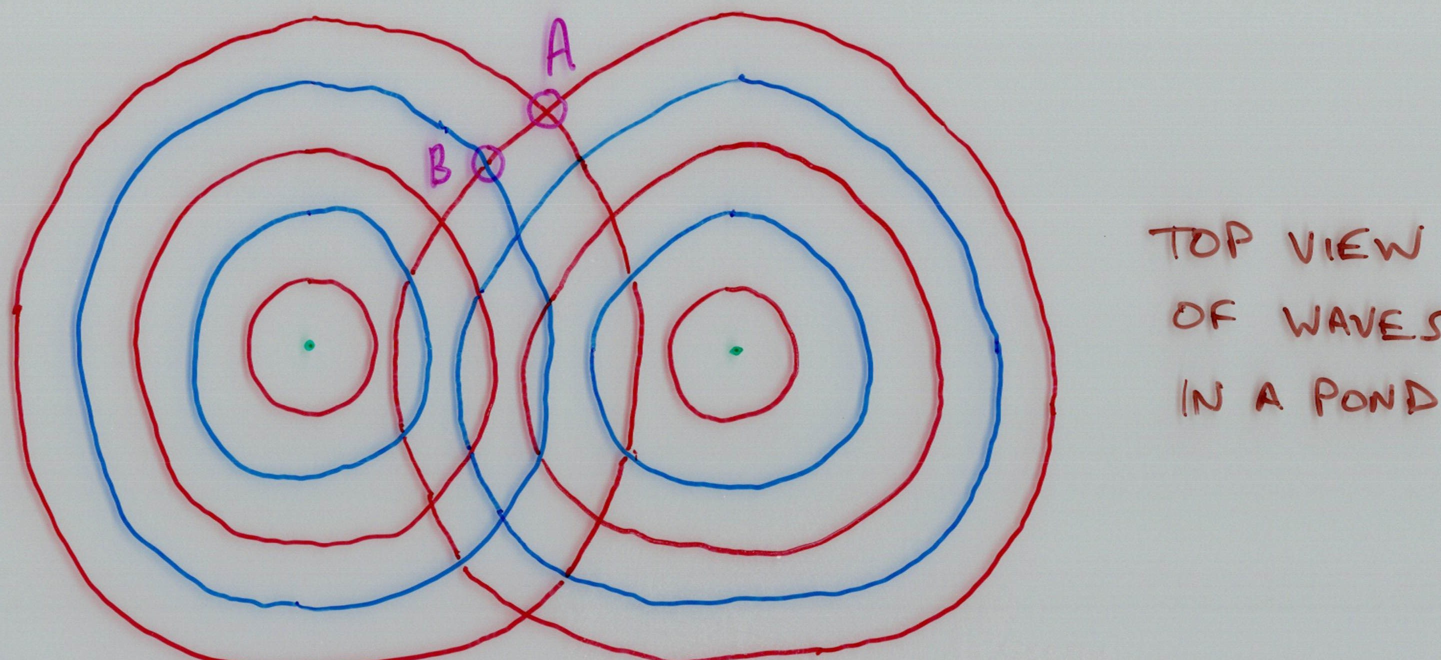


$$P_{12} = P_1 + P_2$$

An Experiment with Waves



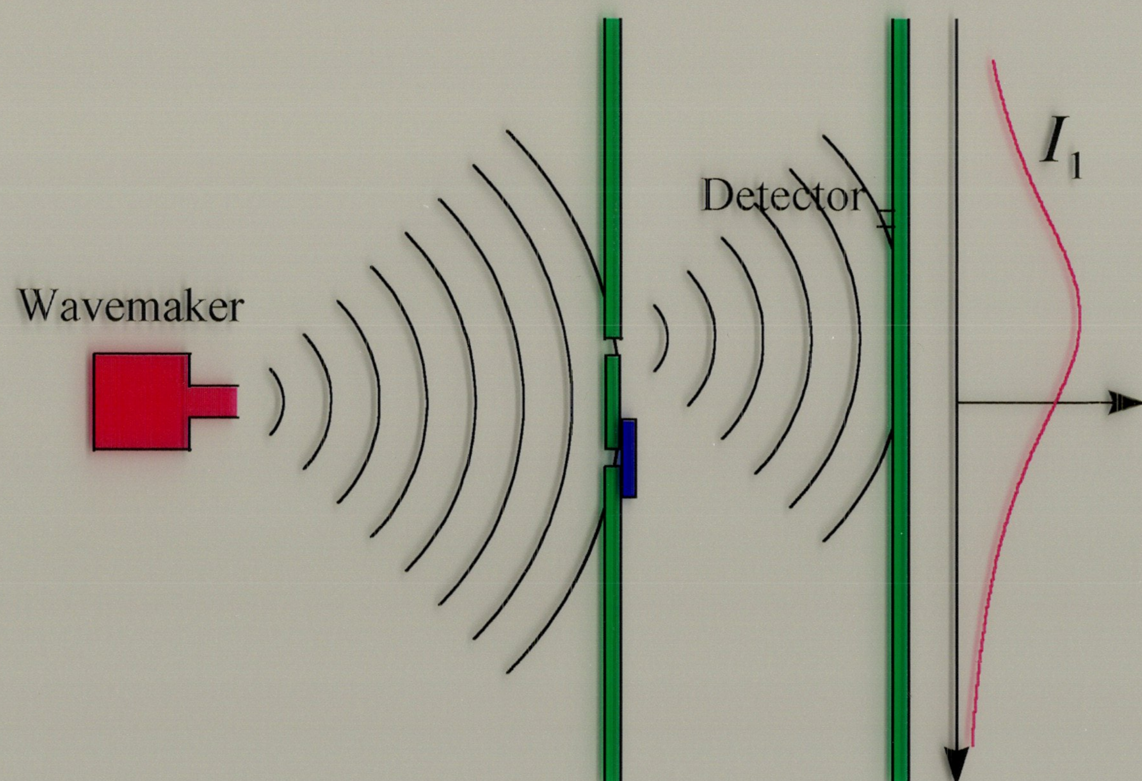




A = CONSTRUCTIVE INTERFERENCE

B = DESTRUCTIVE INTERFERENCE

Wave Experiment with Slit 2 Blocked



Interference Mathematics

$I_1 = |h_1|^2$ with slit 2 closed h_1, h_2 complex

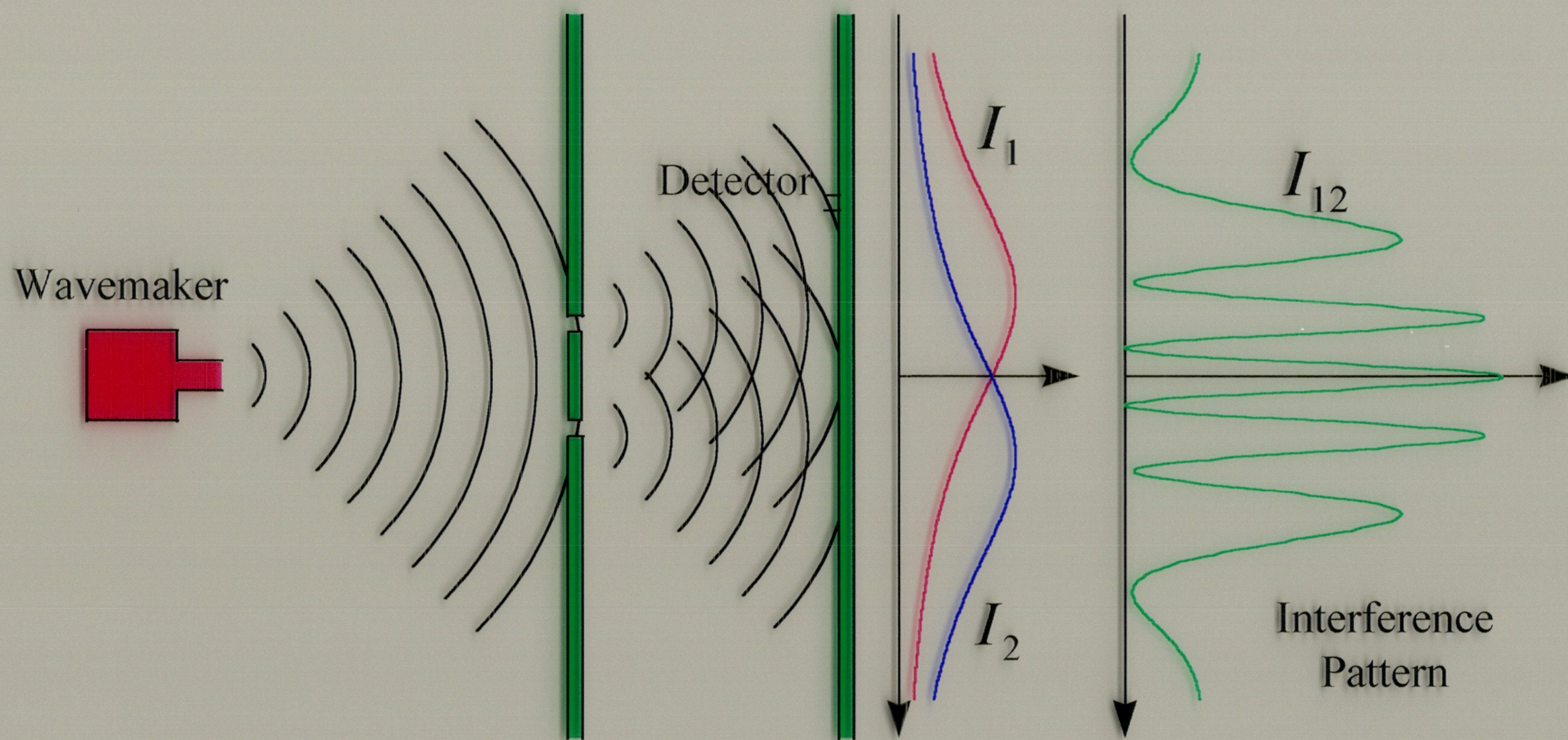
$I_2 = |h_2|^2$ with slit 1 closed

$$I_{12} = |h_1 + h_2|^2 = |h_1|^2 + |h_2|^2 + 2|h_1||h_2|\cos\delta$$

$$= I_1 + I_2 + \underbrace{2\sqrt{I_1 I_2}\cos\delta}_{\text{interference term}}$$

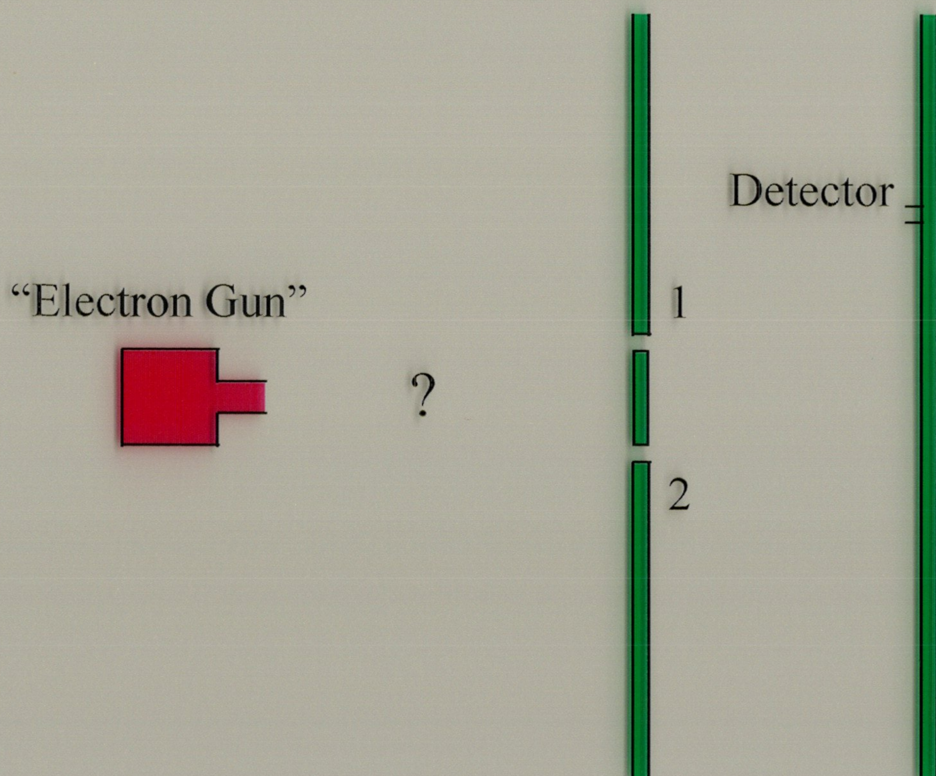
$$(a+b)^2 \neq a^2 + b^2 \qquad (a+b)^2 = a^2 + b^2 + 2ab$$

Wave Experiment: Summary



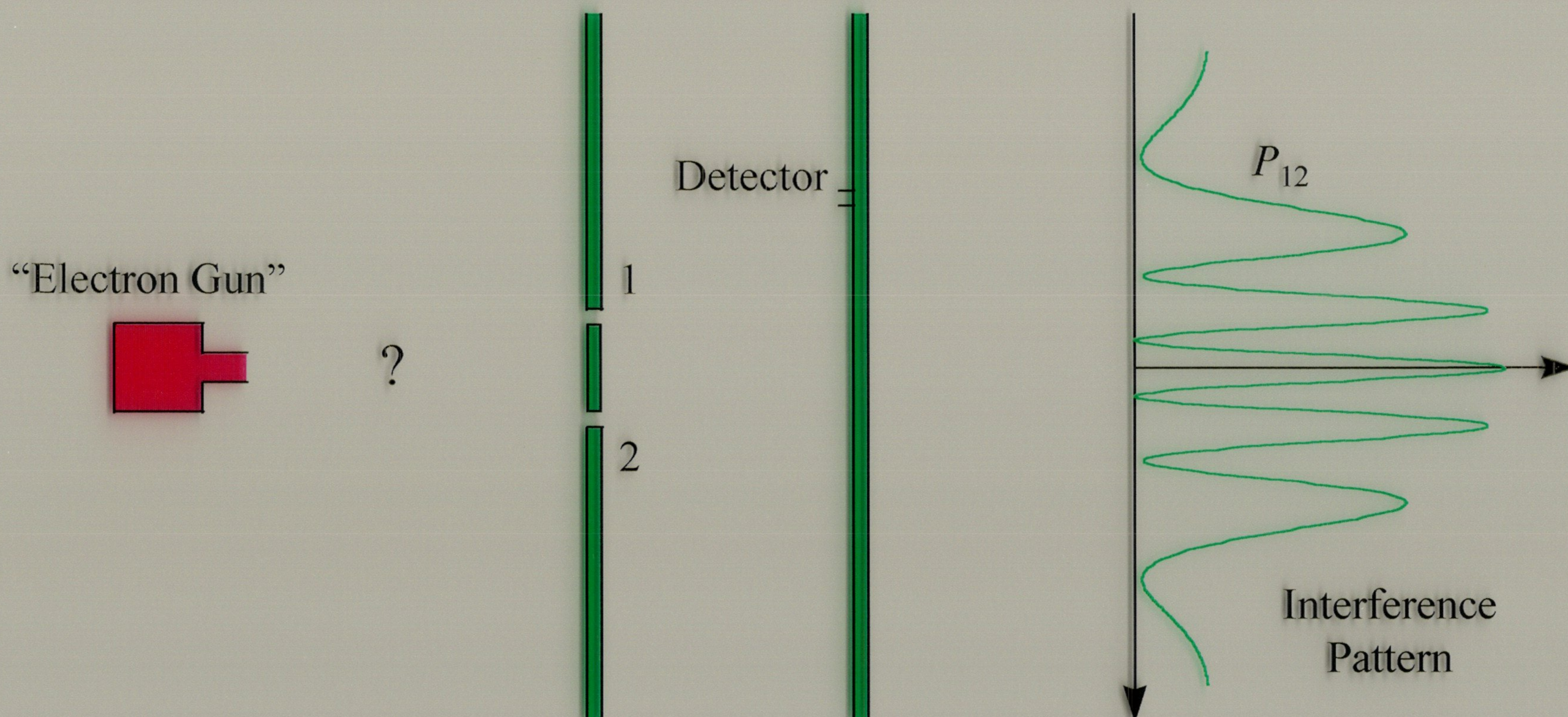
$$I_{12} \neq I_1 + I_2$$

An Experiment with Electrons



•Detector always gives a full click
•No partial clicks
•Electrons arrive in lumps

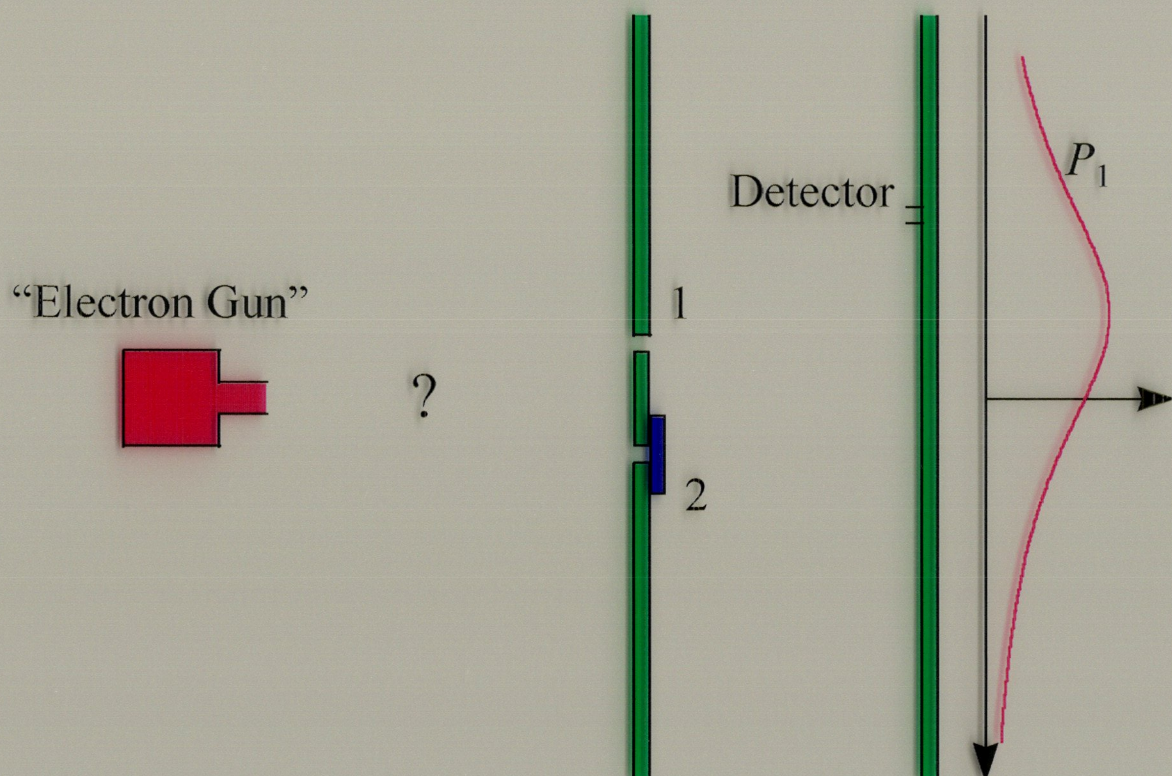
An Experiment with Electrons



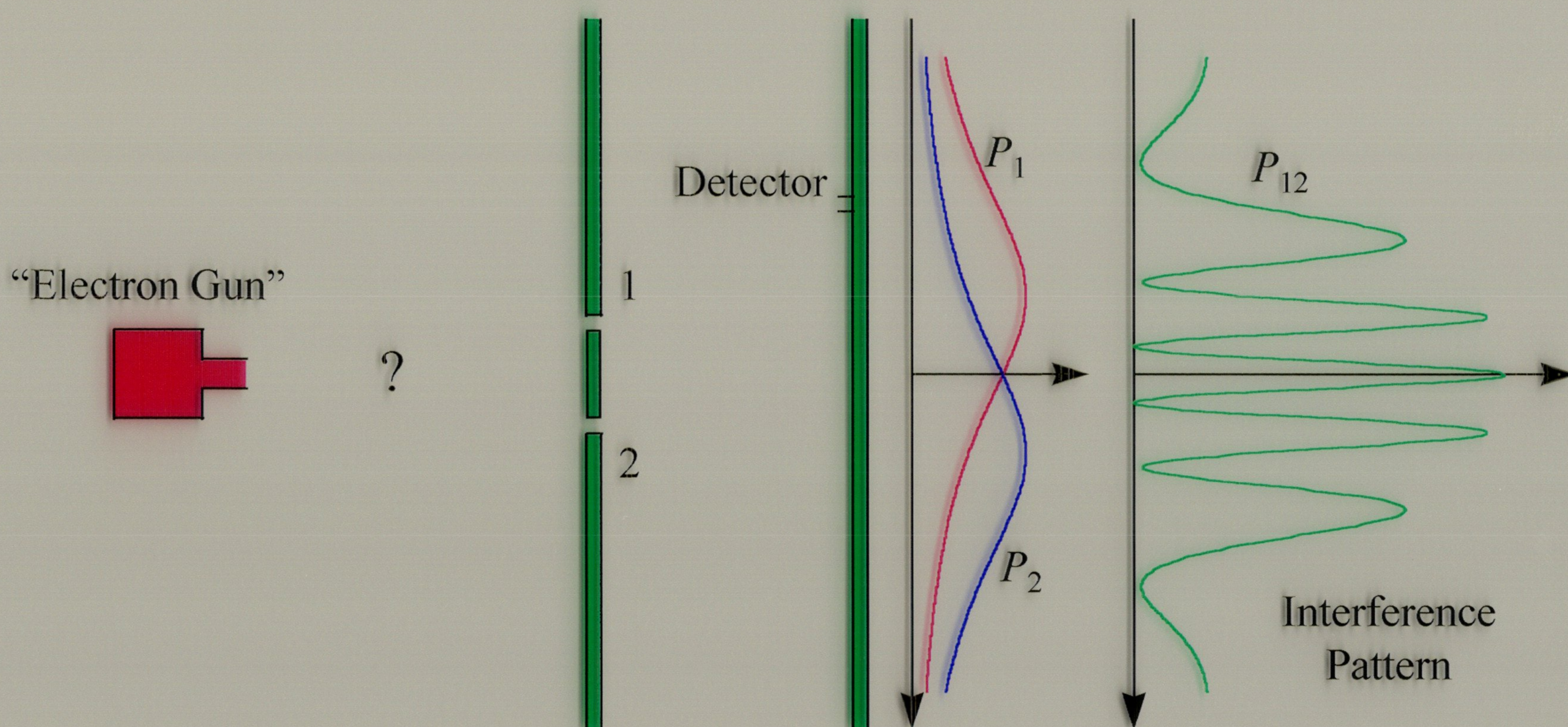
Proposition M

Each electron
either
goes through slit 1
or
goes through slit 2

Electron Experiment with Slit 2 Blocked



Electron Experiment: Summary



$$P_{12} \neq P_1 + P_2$$

Conclude:
Proposition M is false

Interference Mathematics for Electrons is Identical to that for Waves

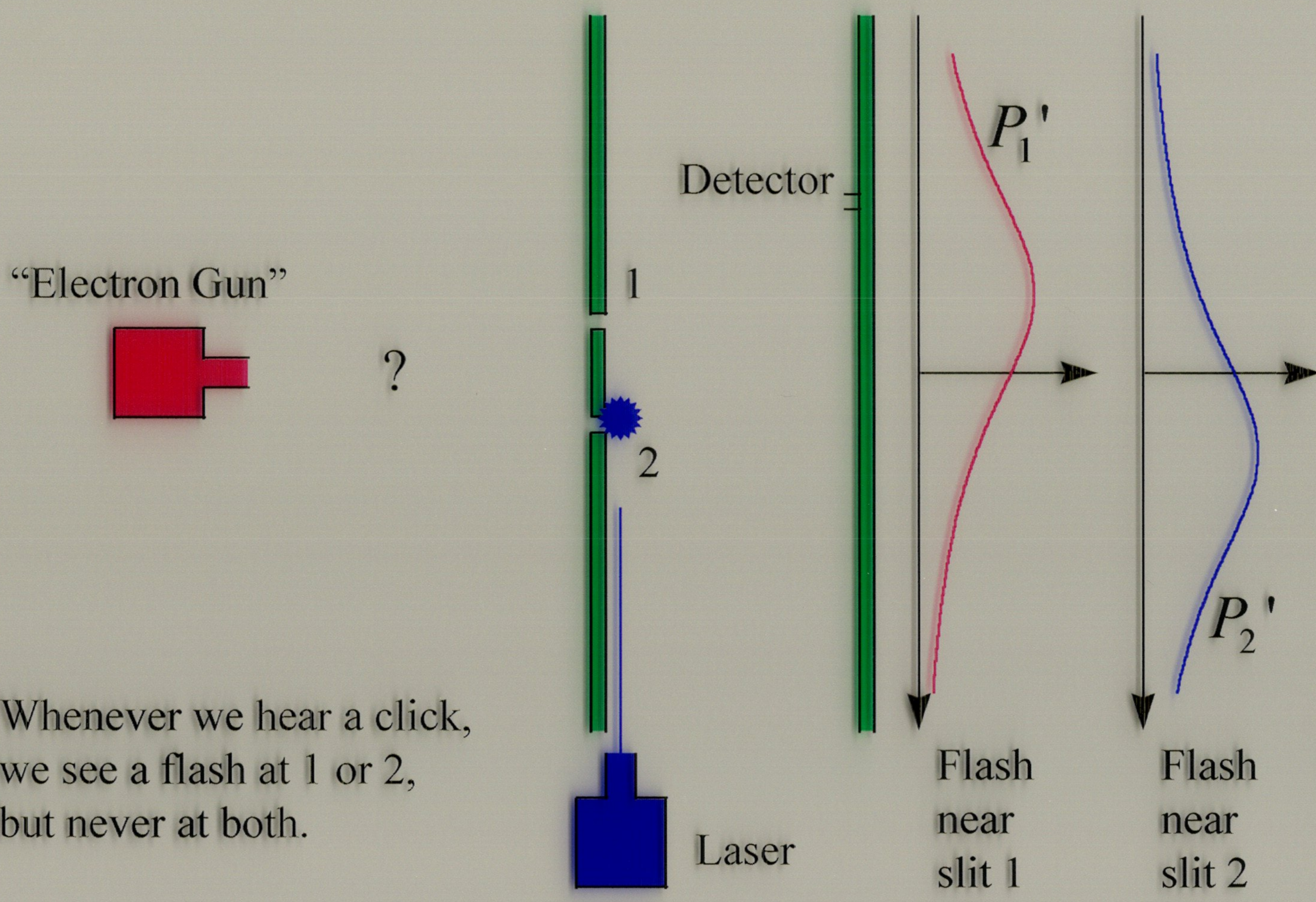
Introduce probability amplitudes φ_1, φ_2

$P_1 = |\varphi_1|^2$ with slit 2 closed φ_1, φ_2 complex

$P_2 = |\varphi_2|^2$ with slit 1 closed

$$P_{12} = |\varphi_1 + \varphi_2|^2 = |\varphi_1|^2 + |\varphi_2|^2 + 2|\varphi_1||\varphi_2|\cos\delta$$
$$= I_1 + I_2 + \underbrace{2\sqrt{I_1 I_2}\cos\delta}_{\text{interference term}}$$

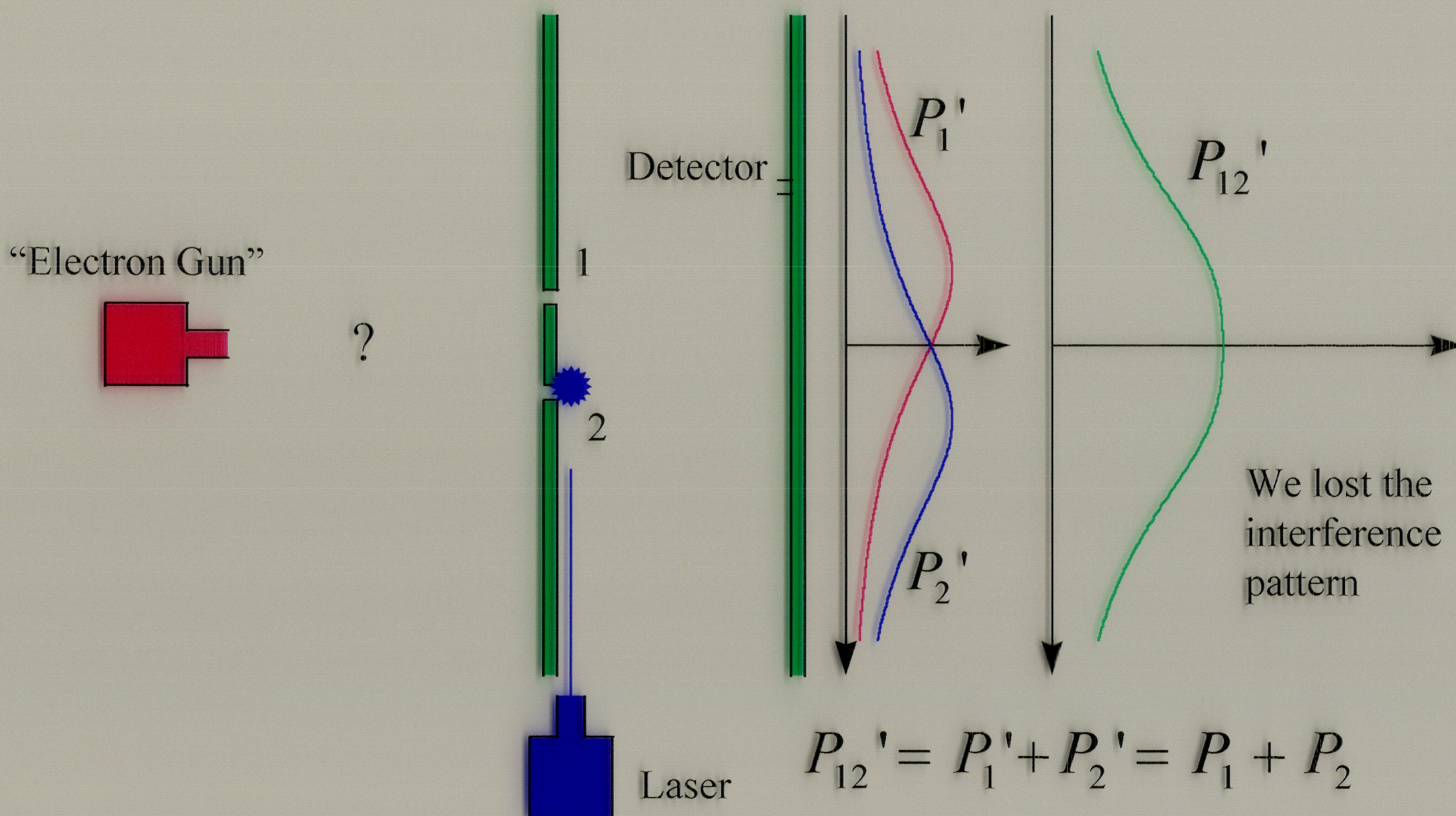
Electron Experiment with Laser



Whenever we hear a click, we see a flash at 1 or 2, but never at both.

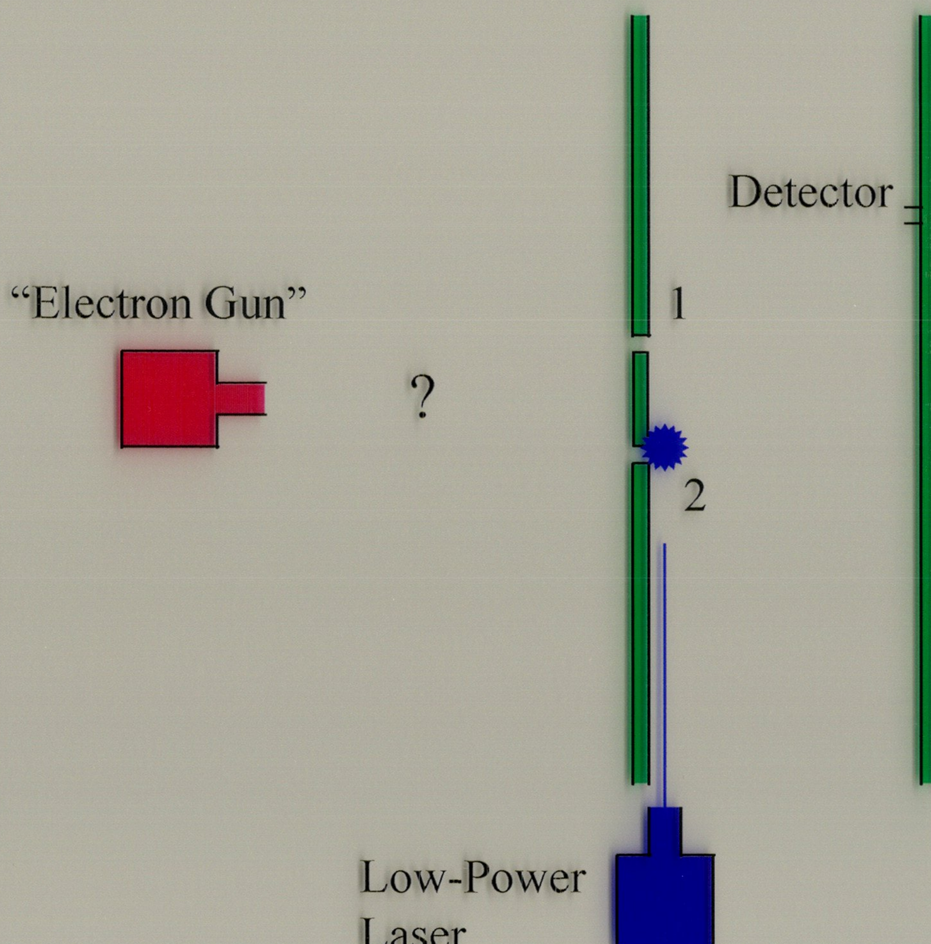
Experimentally: Proposition M is true

Electron Experiment with Laser



$$P_{12}' = P_1' + P_2' = P_1 + P_2$$

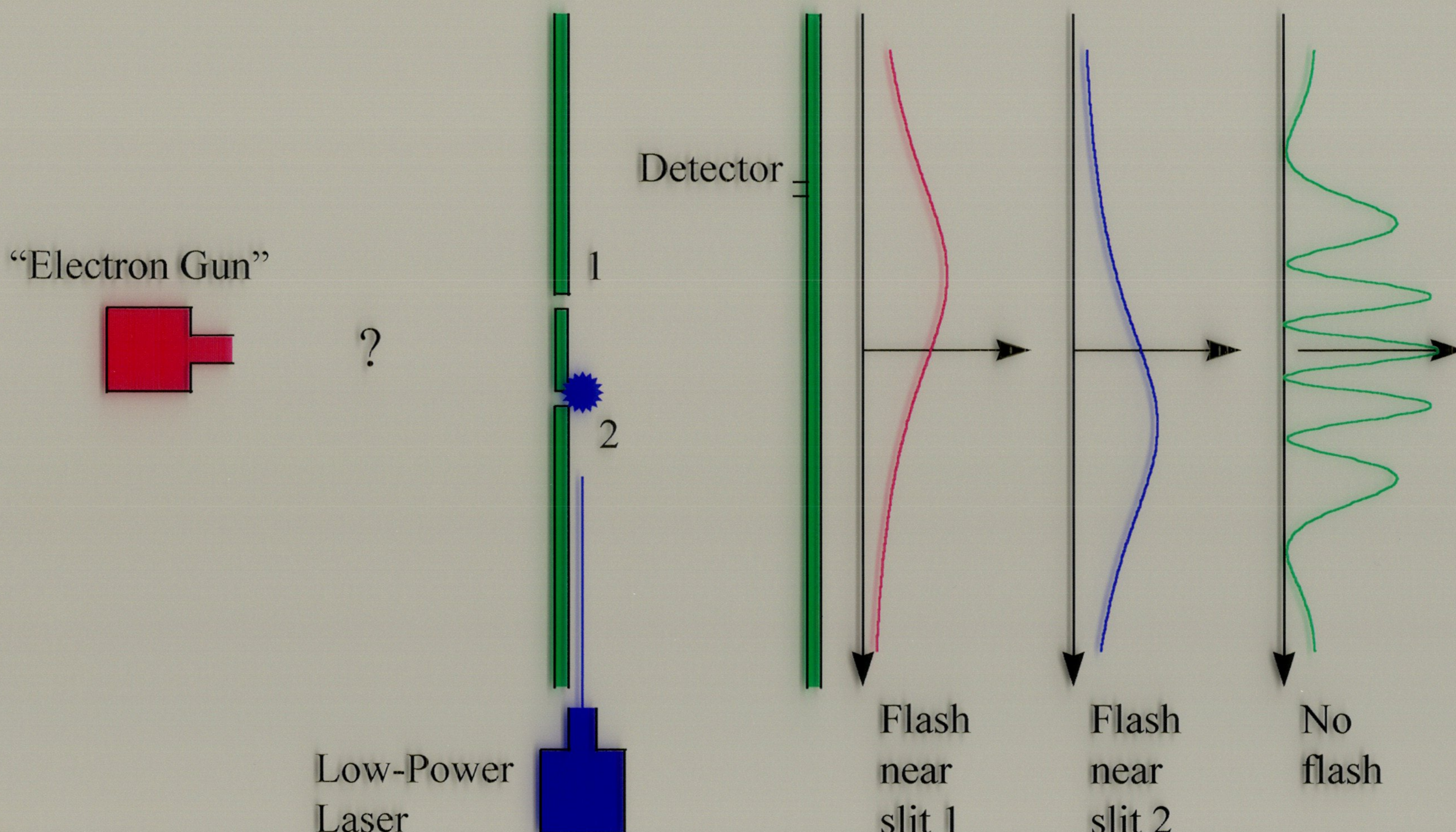
Electron Experiment
with Low-Power Laser



- Always see same sized flash
- Sometimes, click but no flash

- Tally results:
  - Flash near 1
  - Flash near 2
  - No flash

Electron Experiment
with Low-Power Laser



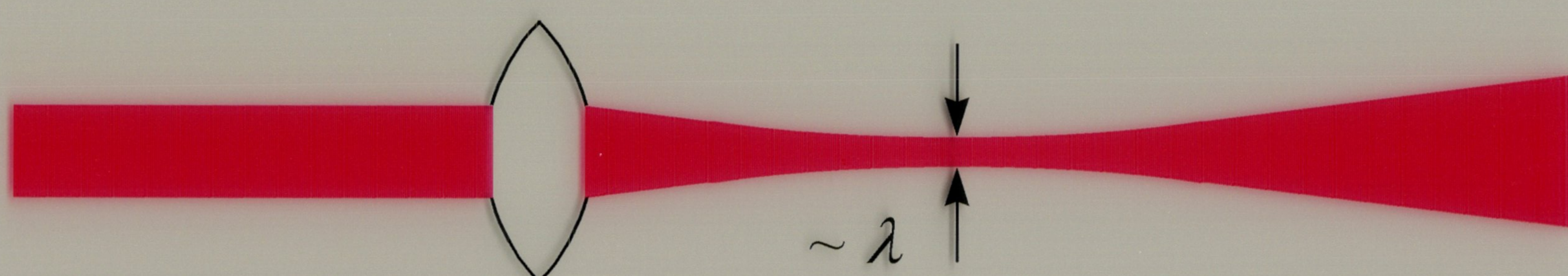
A few more properties of light

Momentum of a photon is inversely proporional to its wavelength

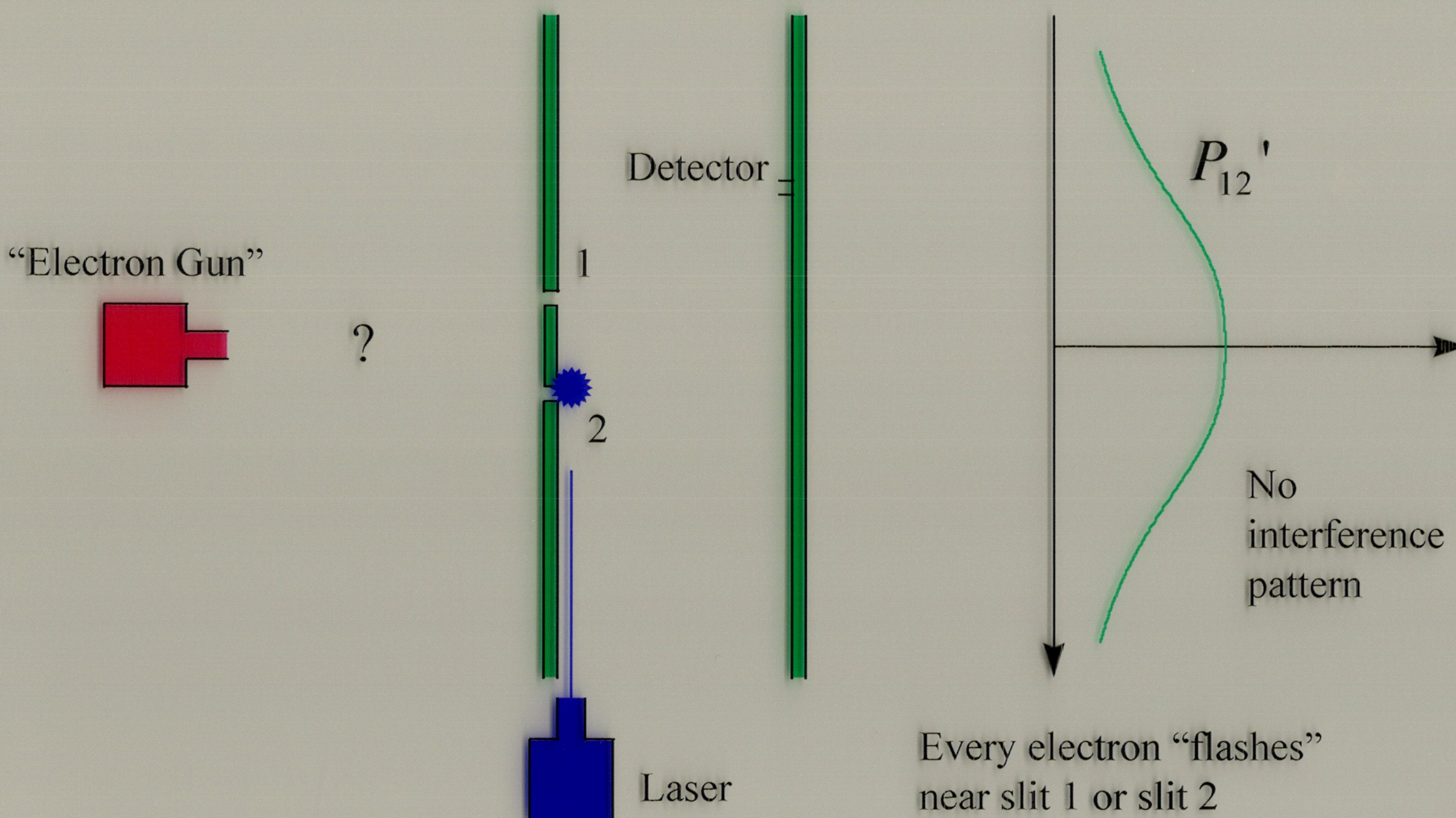
$$p = \frac{h}{\lambda} = \frac{h}{c}\nu$$

Wavelength   Frequency

Can’t focus light to a point:

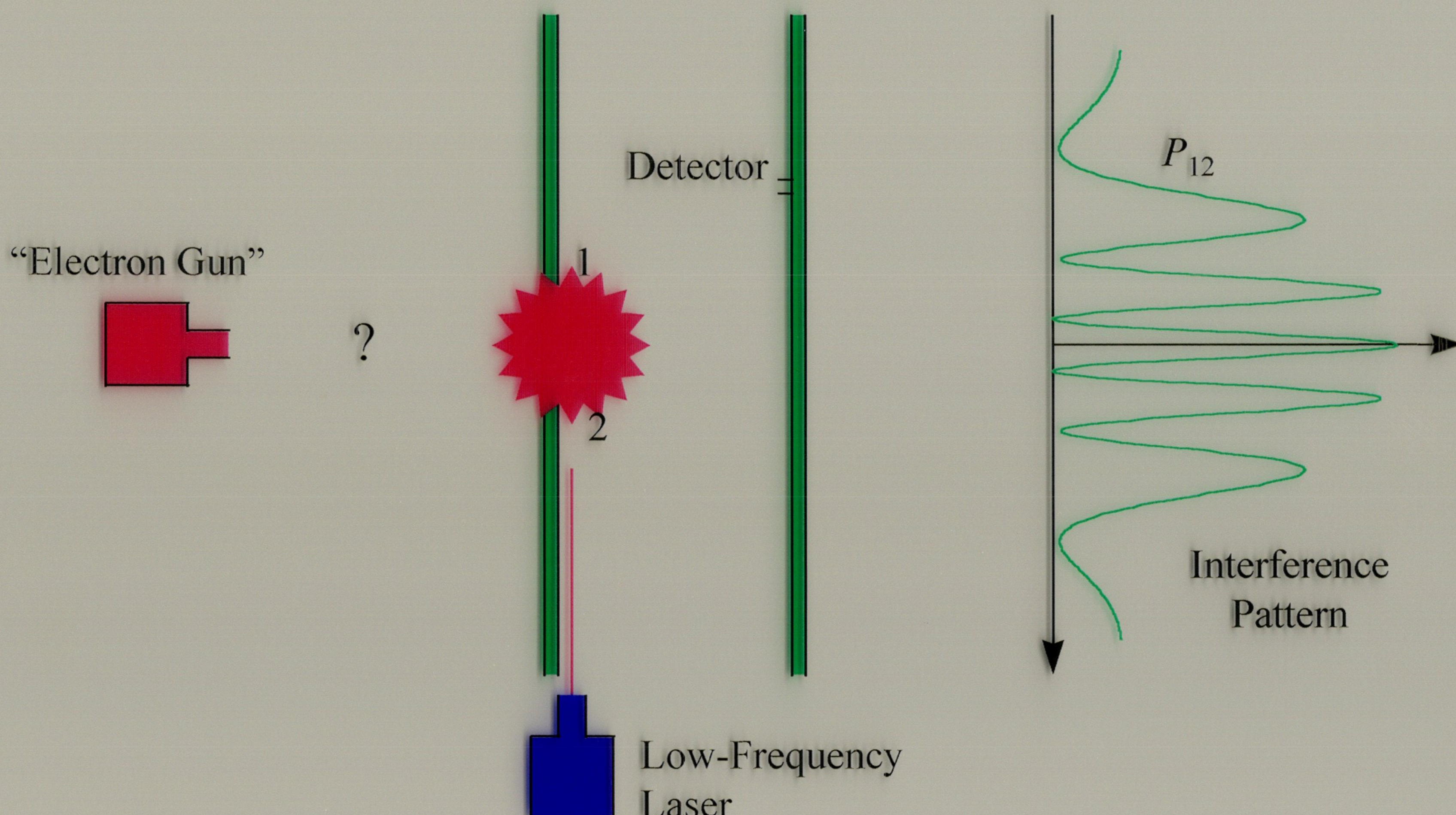


We'll try to get the interference pattern back by lowering the frequency of the light



Every electron "flashes" near slit 1 or slit 2

The interference pattern returns when we can't resolve which path the electron took

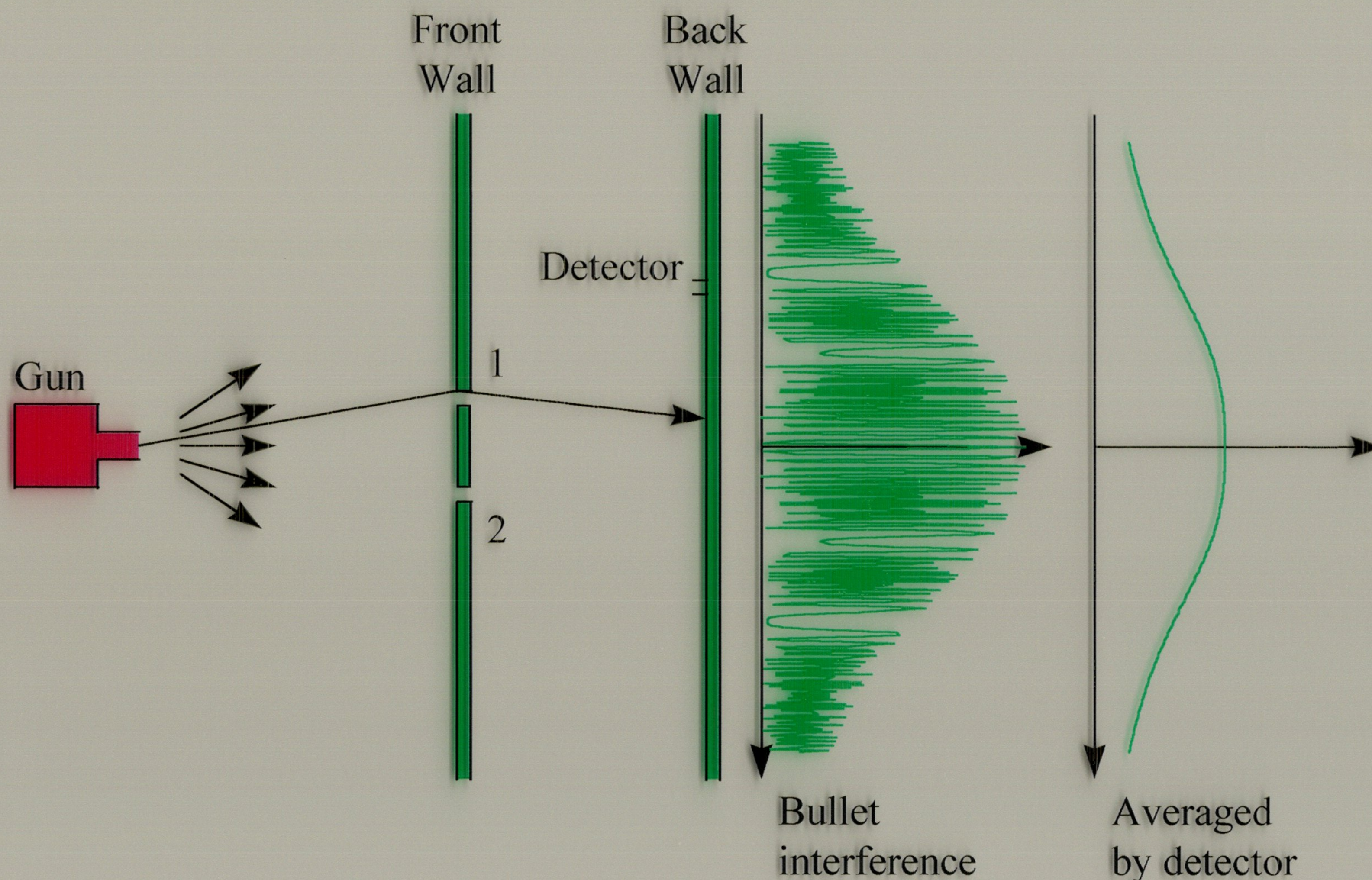


What about Proposition M?

If one can determine experimentally which slit an electron went through, then Proposition M is true.

If one cannot determine which slit an electron went through, then we don’t know.

Quantum Mechanics of Bullets



Basic Rules of Quantum Mechanics

1. Probability of an event is given by the square of a probability amplitude

$$P = |\varphi|^2$$

2. If an event can occur in several ways, add amplitudes

$$\varphi = \varphi_1 + \varphi_2 \qquad\qquad P = |\varphi_1 + \varphi_2|^2$$

3. If an experiment is performed which can determine the way taken, add probabilities

$$P = P_1 + P_2$$

Conclusions

- Electrons (and other small particles) are not classical particles and are not waves.
- "Wave-Particle Duality" refers to the fact that sometimes they behave like each.
- Though mathematically sound and experimentally upheld, Quantum Mechanics remains interpretationally precarious.