Quantum Mysteries

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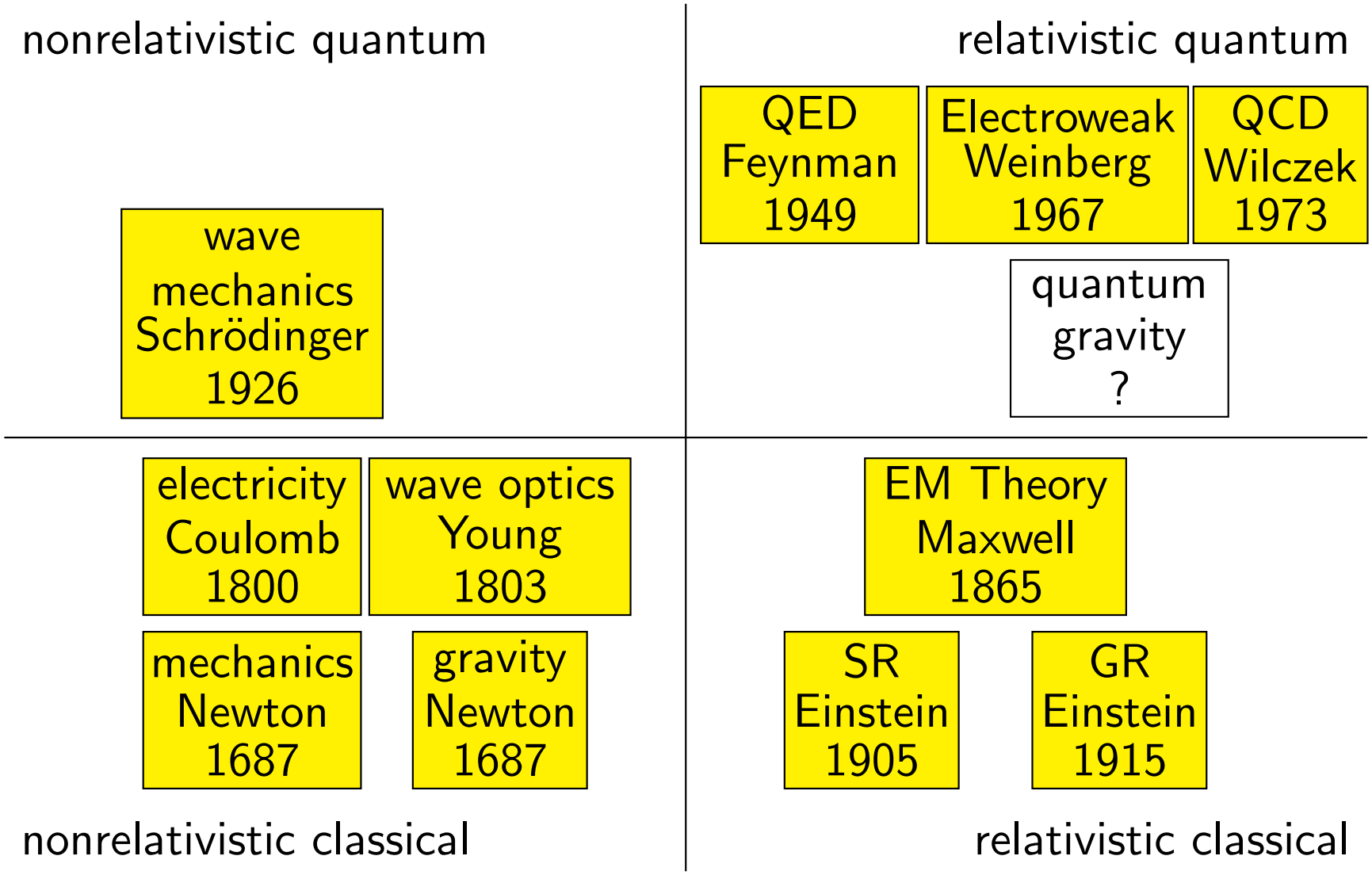
Key events in the development of Quantum Theory

1900	Planck proposes quanta of light
1905	Einstein explains photoelectric effect
1913	Bohr suggests special radii
1921	Stern and Gerlach demonstrate spatial quantization
1923	Compton sees frequency shift in scattered X-rays
1924	de Broglie suggests matter waves
1925	Heisenberg presents matrix mechanics
1926	Schrödinger presents wave mechanics
1927	Heisenberg presents uncertainty principle

Theories in Physics



Metric (SI) units

Dimension	Symbol	Unit	Abbreviation
Length	x	meter	m
Mass	m	kilogram	kg
Time	t	second	s
Speed	v		m/s
Energy	E	Joule	J $=$ kg m^2/s^2
Momentum	p		kg m/s
Angular Momentum	L		J s $=$ kg m^2/s
Frequency	f	Hertz	Hz $=$ cycle/s

Metric prefixes

Prefix	Symbol	Meaning
Tera	T	10^{12}
Giga	G	10^{9}
Mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}

- $3\ \text{nm} = 3 \times 10^{-9}\ \text{m} = 0.000000003\ \text{m}$

Unit conversion

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{J}$$

- Example: Convert 3×10^{-18} J into electron volts.
  Solution:

$$3 \times 10^{-18} \text{ J} \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} = 18.7 \text{ eV}$$

- Example: Convert 1.7 eV into Joules.
  Solution:

$$1.7 \text{ eV} \times \frac{1.602 \times 10^{-19} \text{ J}}{1 \text{ eV}} = 2.72 \times 10^{-19} \text{ J}$$

Classical waves

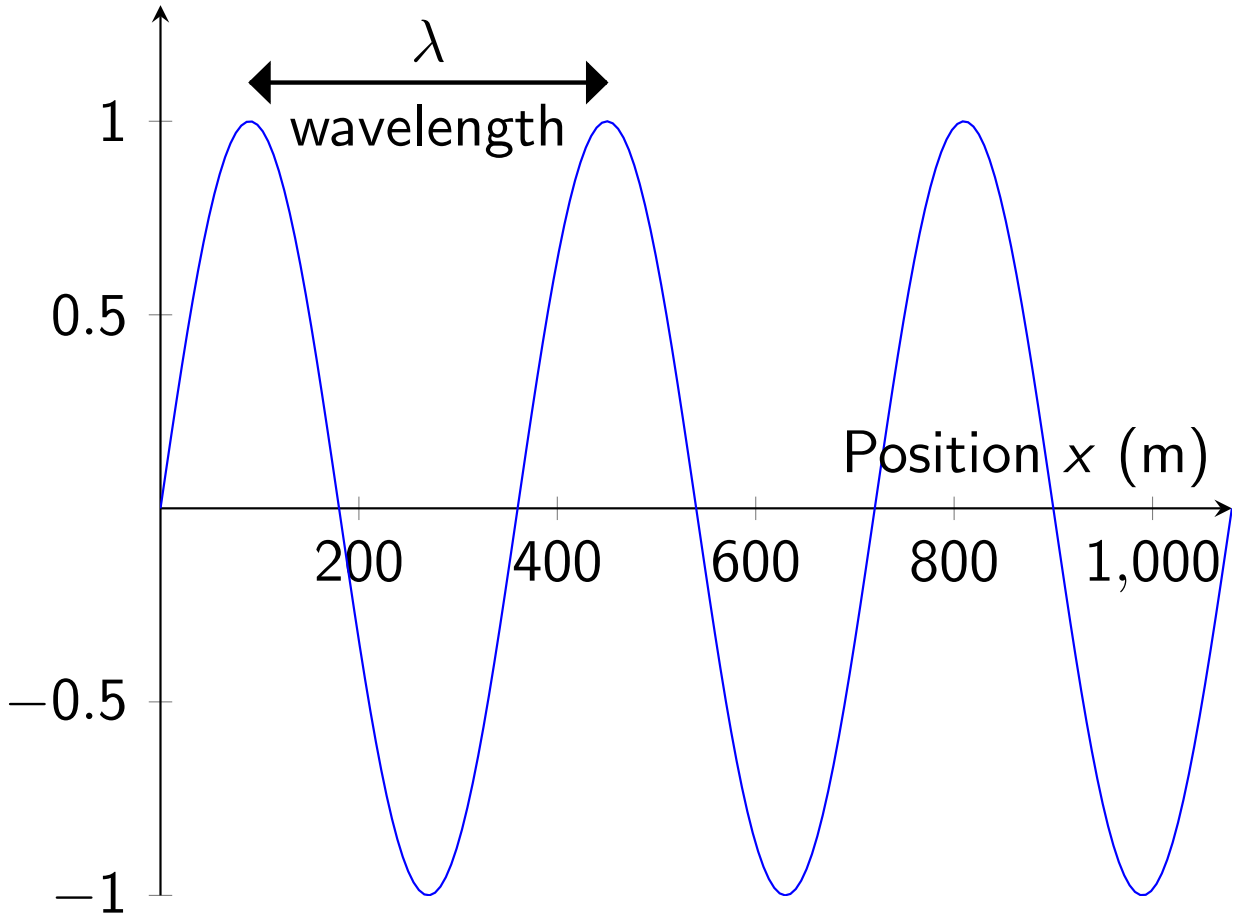
There are many types of waves.

- sound waves
- water waves
- light waves
- waves on a string

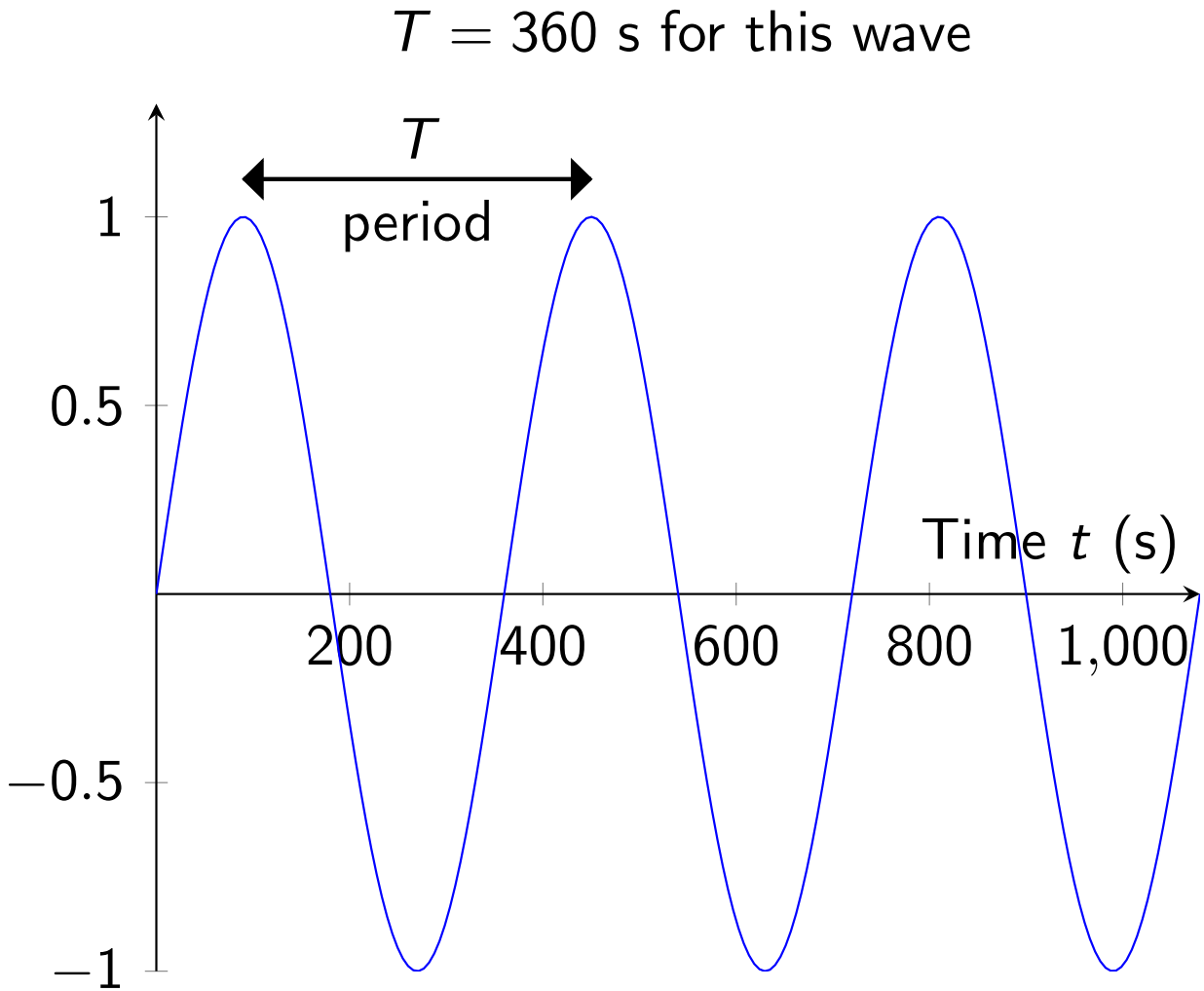
A basic wave (of any type) has 5 important properties.

- *Wavelength* is crest-to-crest distance at a snapshot in time
- *Period* is time between crest arrivals at a fixed point in space
- *Frequency* is the number of crests per second moving past a fixed point
- Wave *speed* is the speed of a crest
- *Amplitude* is the size or strength of a wave

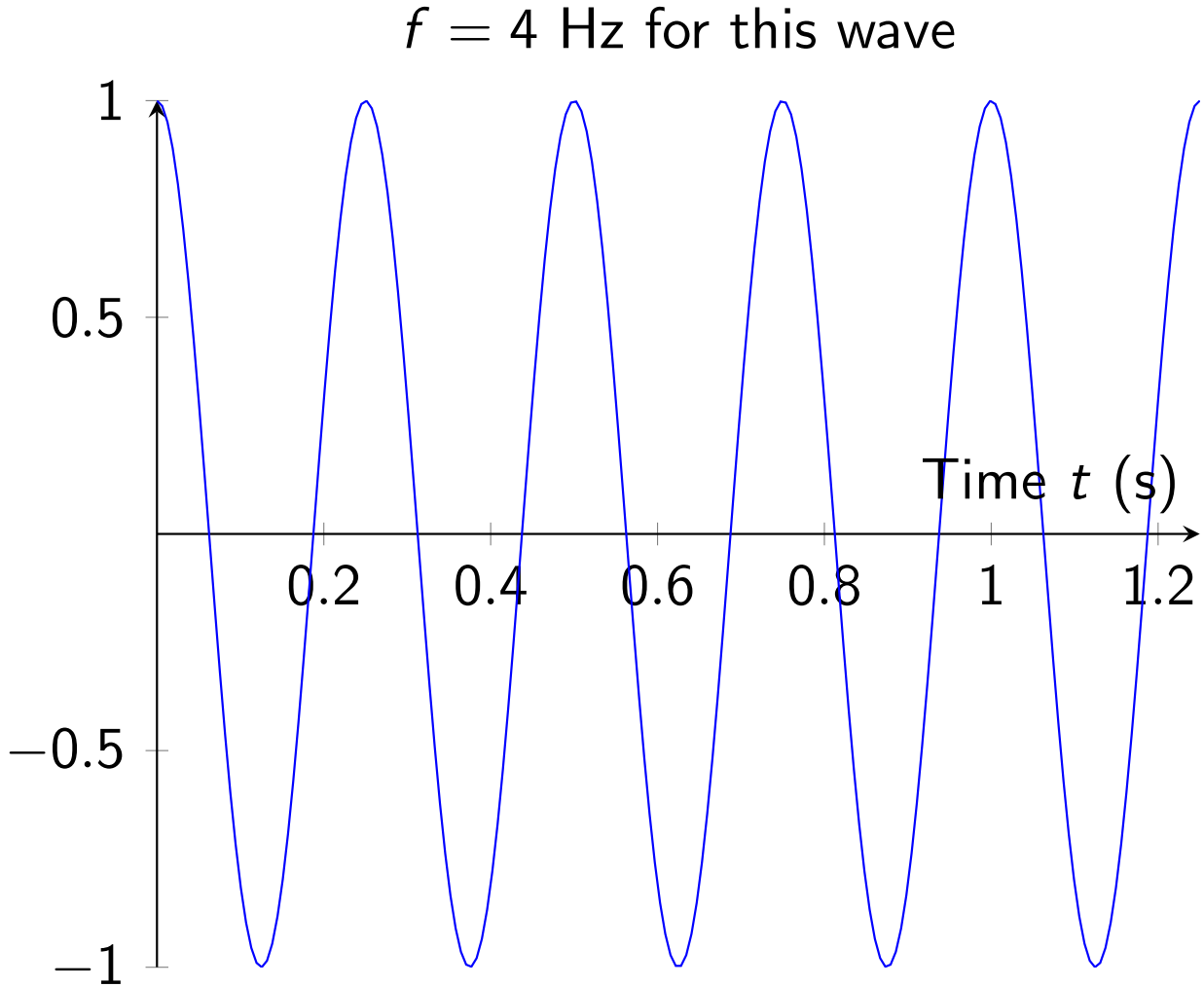
Wavelength is crest-to-crest distance at a snapshot in time



Period is time between crest arrivals at a fixed point in space



Frequency is the number of crests per second moving past a fixed point



The 5 wave properties are not independent

Period and frequency contain the same information in different form.

$$f = \frac{1}{T}$$

Wavelength, period, and speed are related. The wave travels at one wavelength per period.

$$v = \frac{\lambda}{T} = \lambda f$$

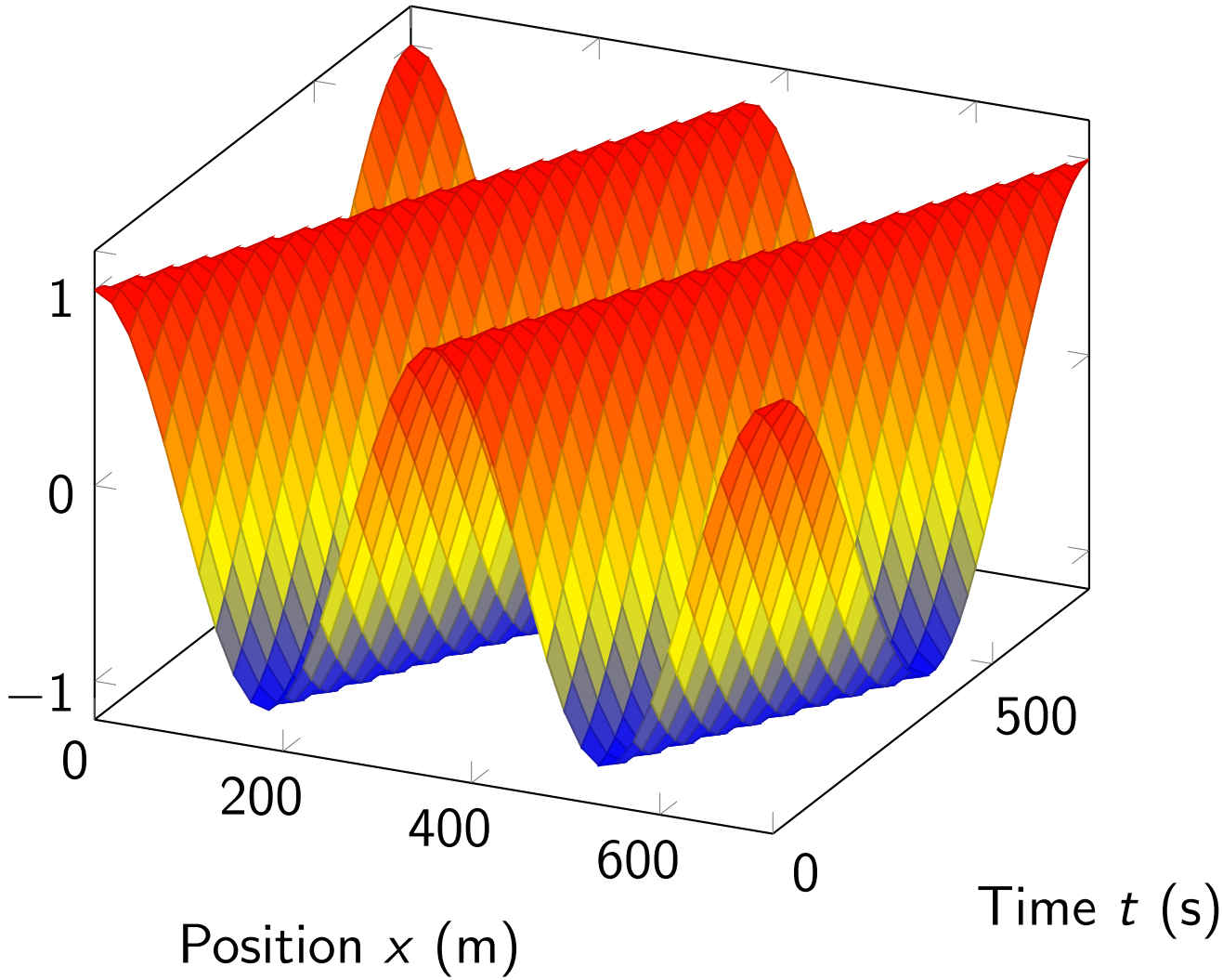
Classical wave summary

Property	Symbol	Unit
Wavelength	λ	m
Period	T	s
Frequency	f	Hz
Speed	v	m/s
Amplitude	depends	depends

$$f = \frac{1}{T}$$

$$v = \frac{\lambda}{T} = \lambda f$$

A wave moves in space and time

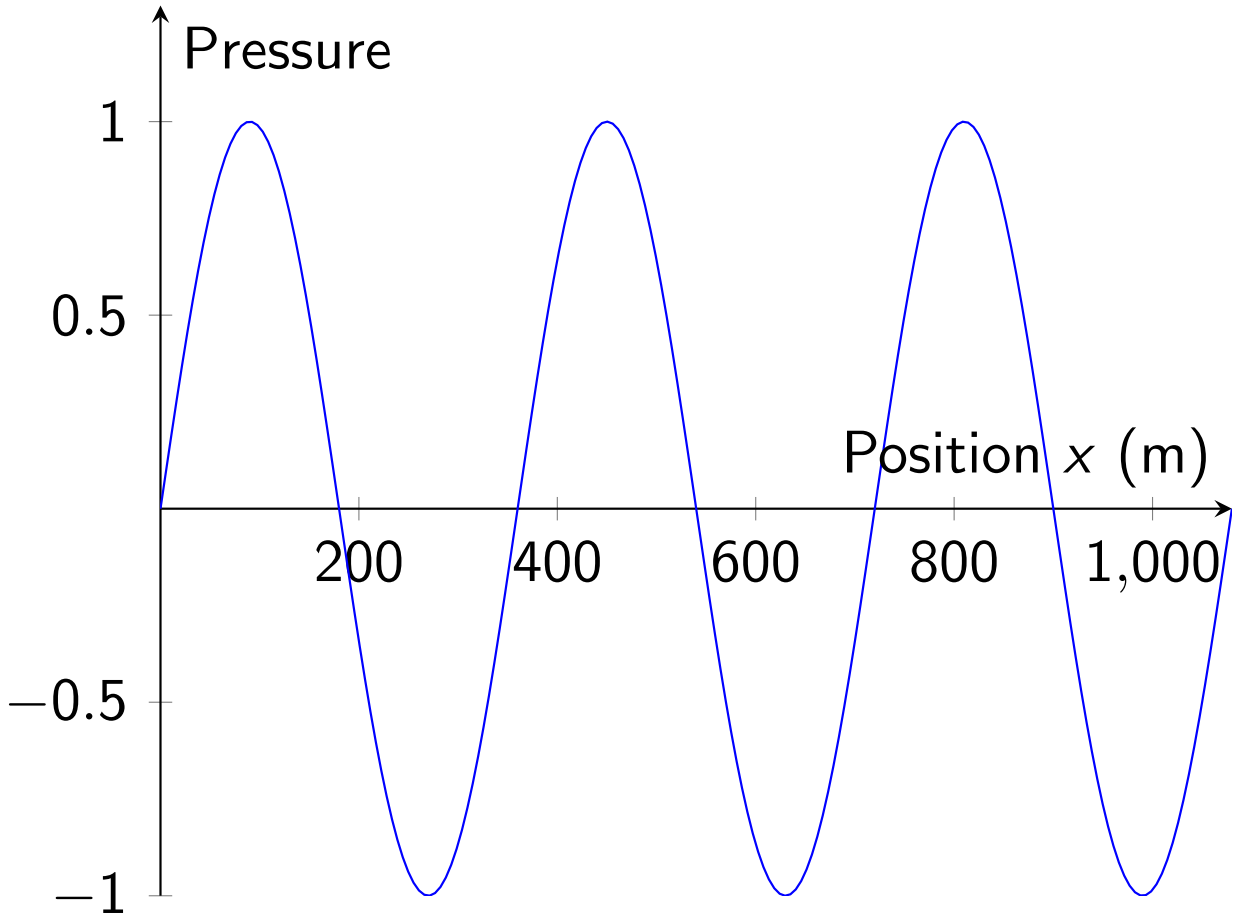


- This wave moves in the positive x direction.

A sound wave is a pressure wave

Speed of sound in air is about 343 m/s (depending on temperature and humidity)

Air pressure oscillates in space and time



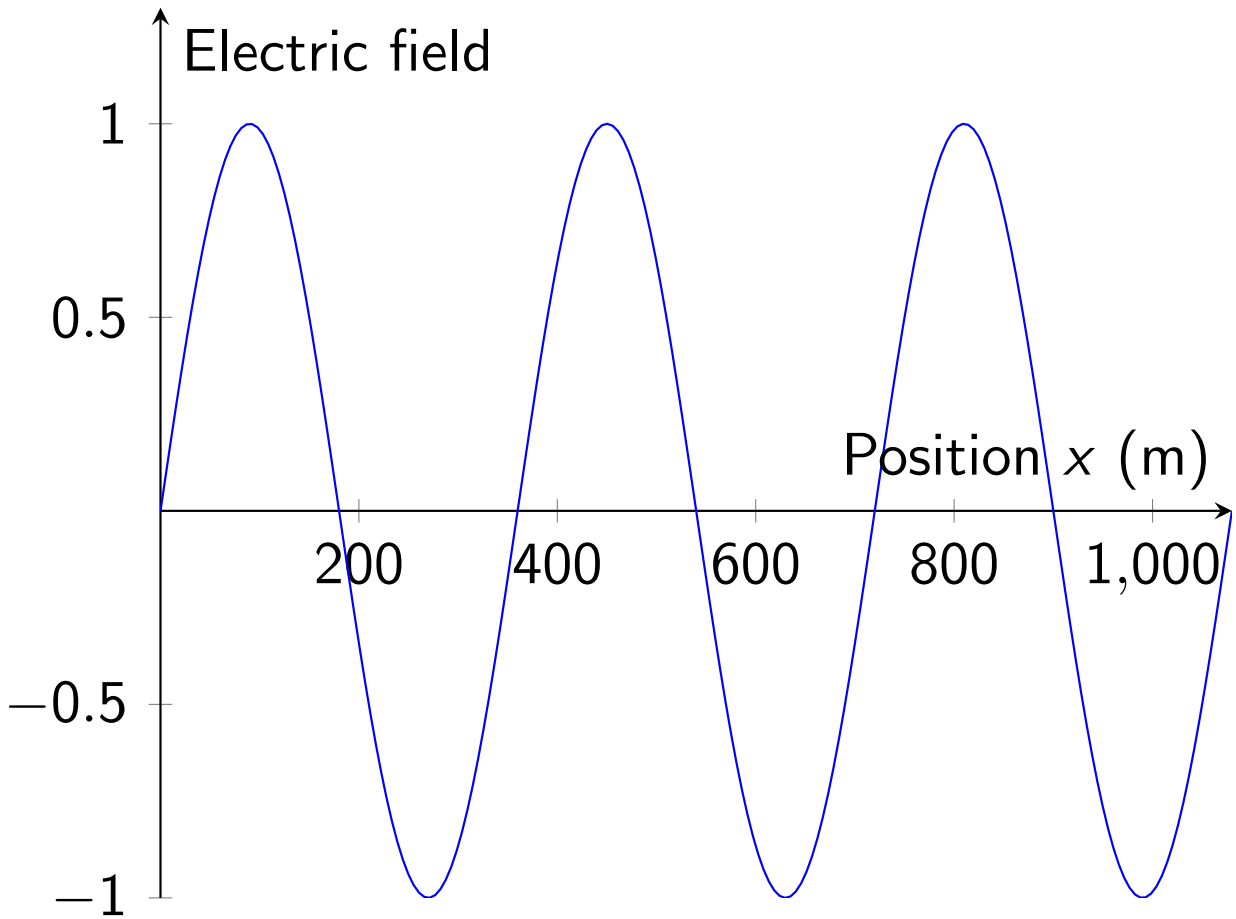
Exercise: Find the wavelength of a sound wave

On the piano, the frequency of the A above middle C is 440 Hz. What is the wavelength of this sound wave as it travels through air?

A light wave is an electromagnetic wave

Speed of light in vacuum is 3×10^8 m/s.

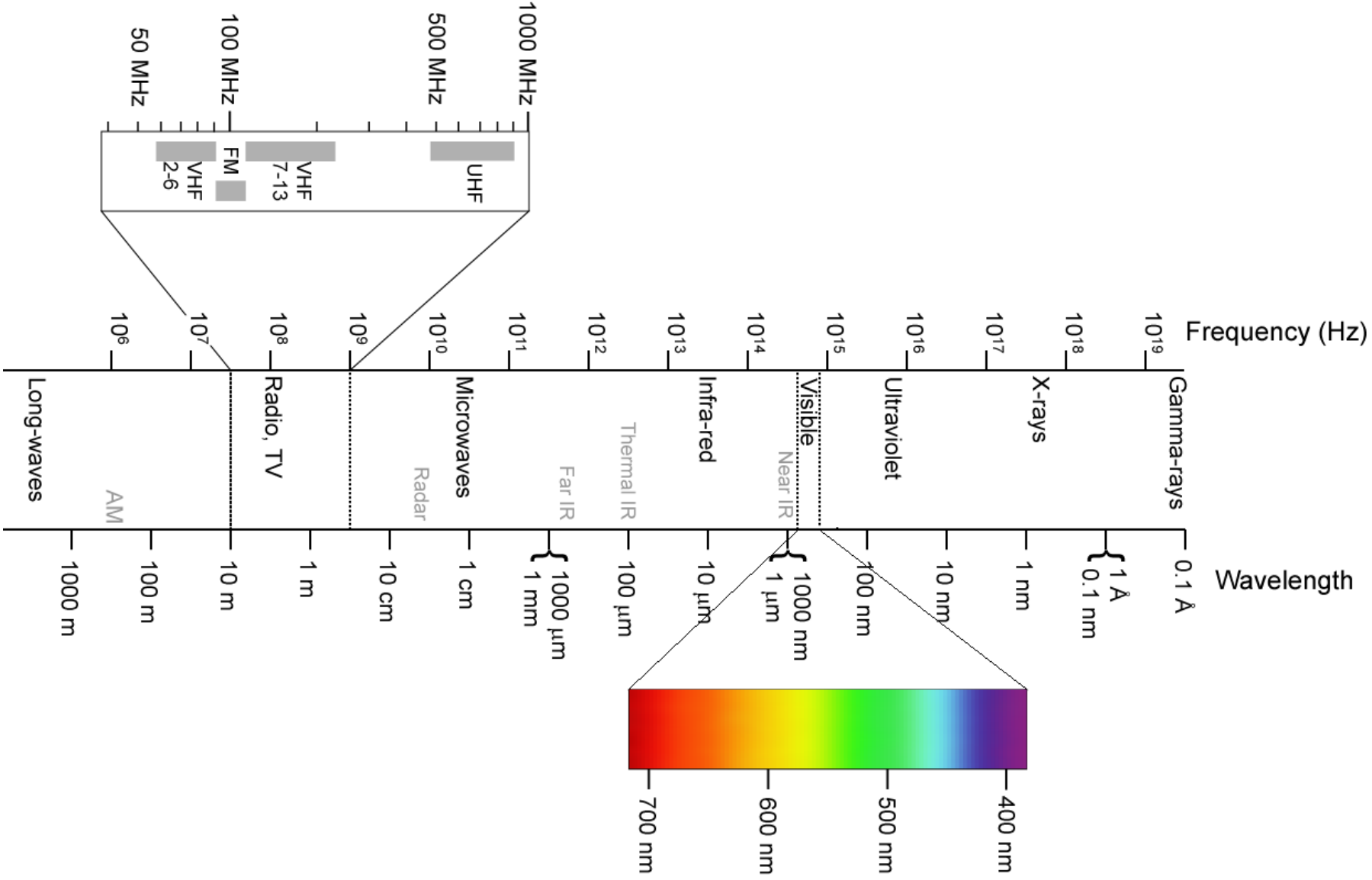
Electric field oscillates in space and time



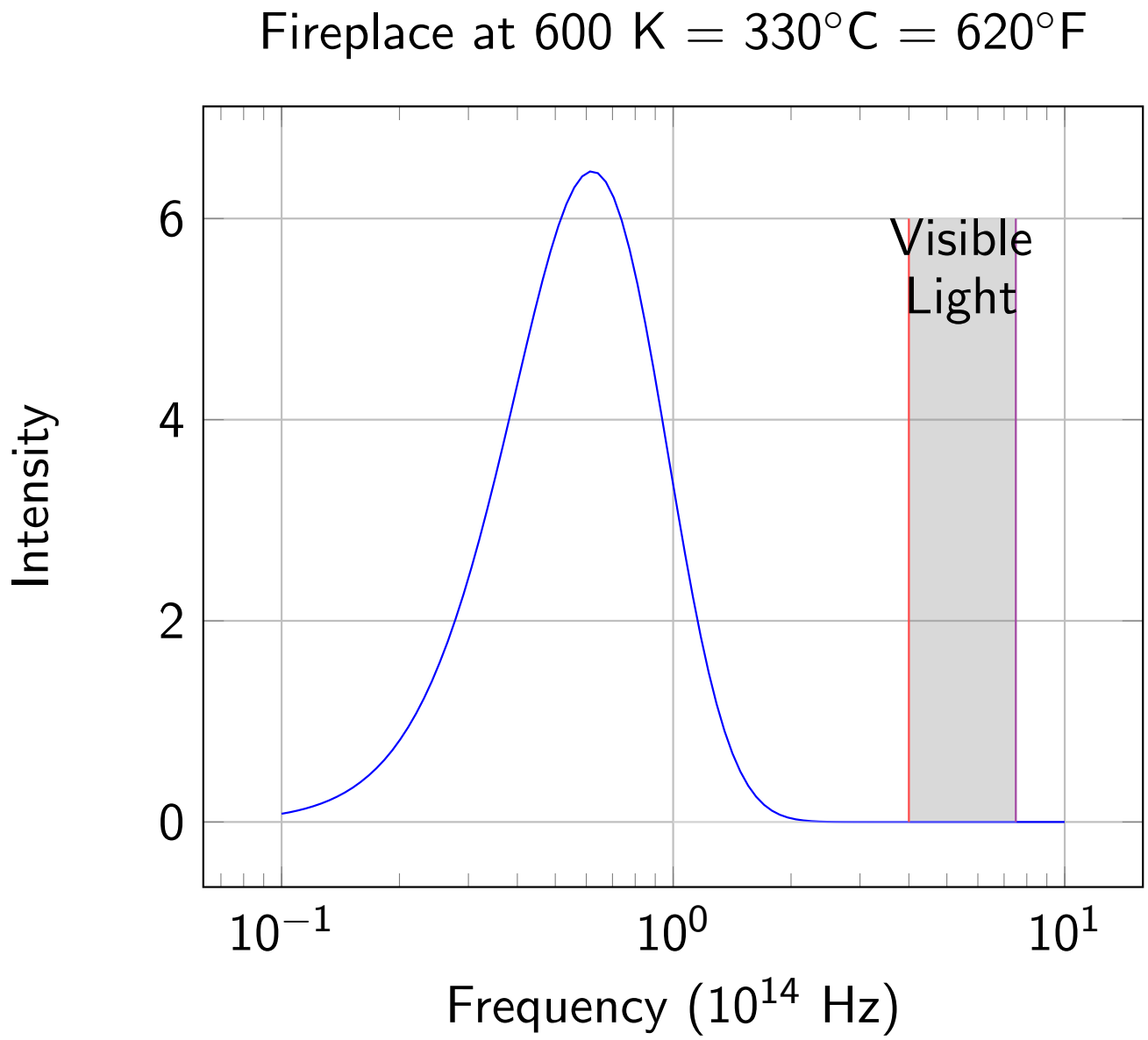
Exercise: Find the frequency of a light wave

Red light has a wavelength of 700 nm. What is its frequency?

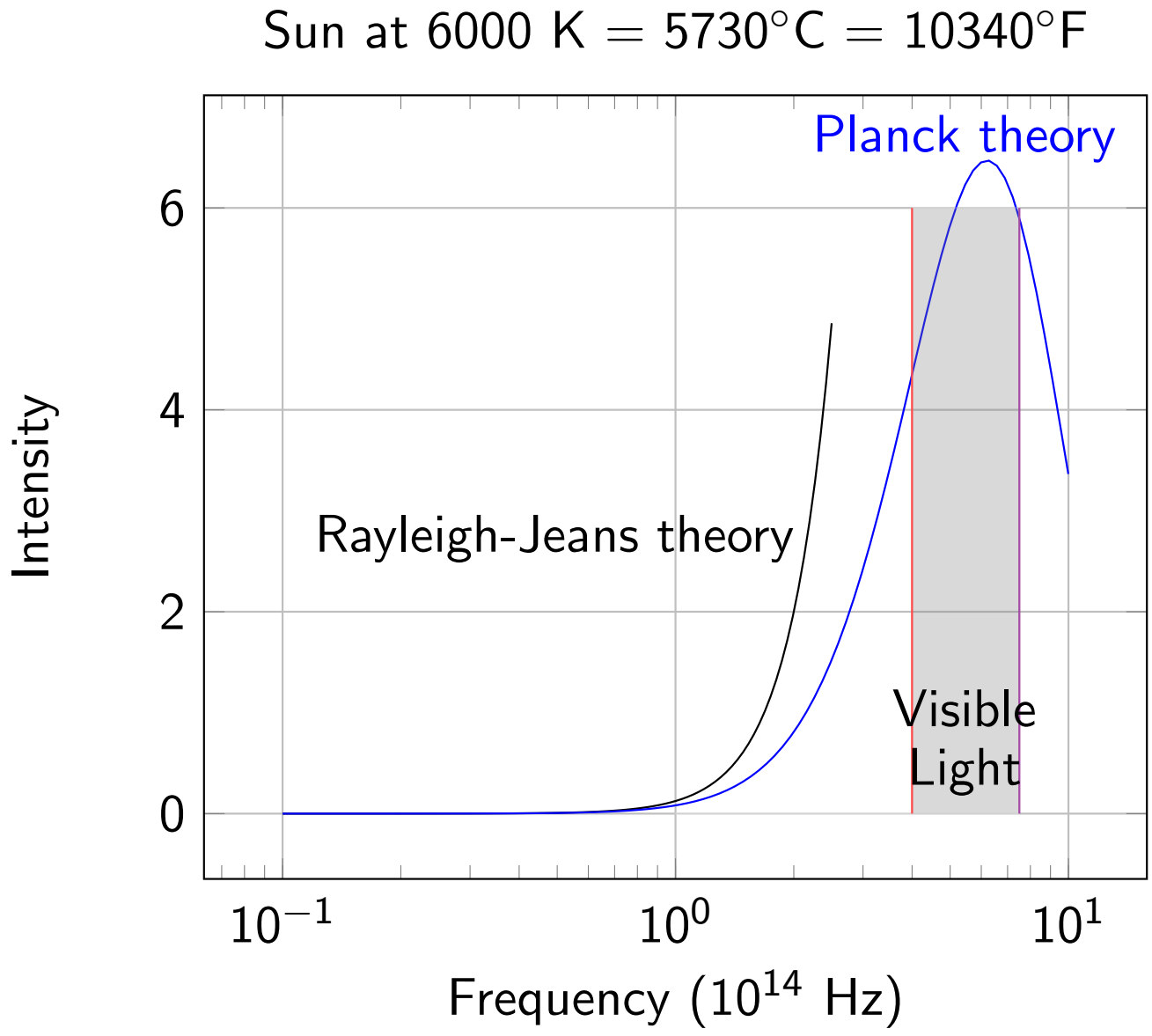
The electromagnetic spectrum



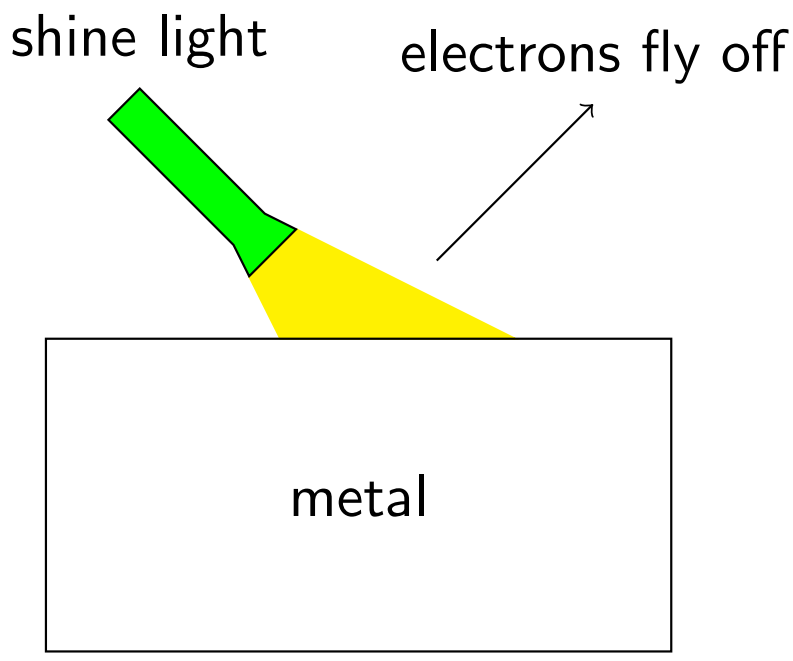
Hot things radiate



The ultraviolet catastrophe

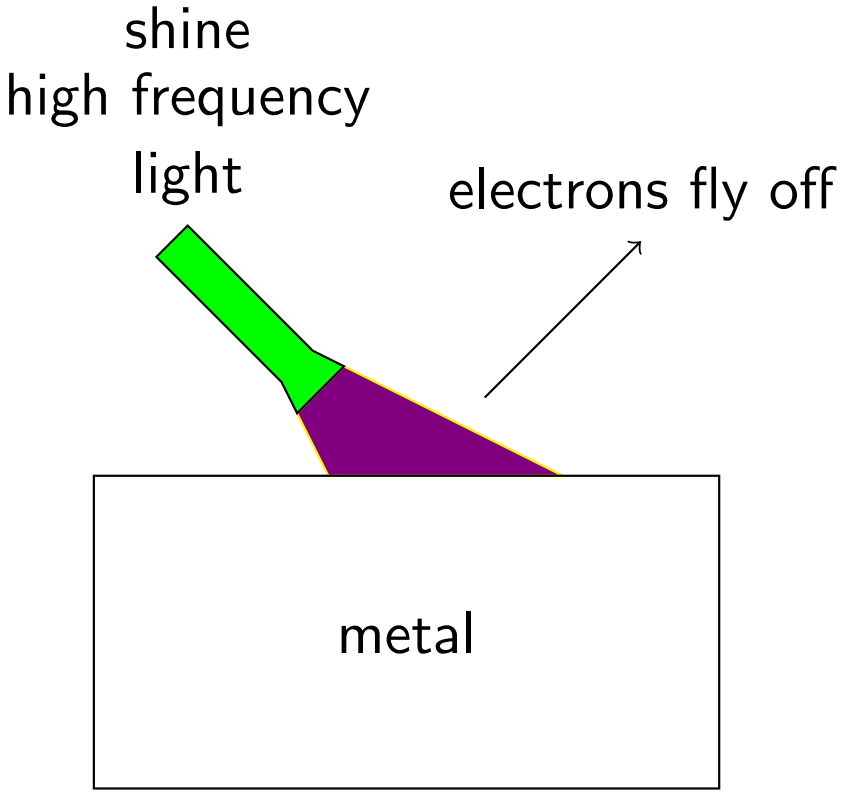


The photoelectric effect

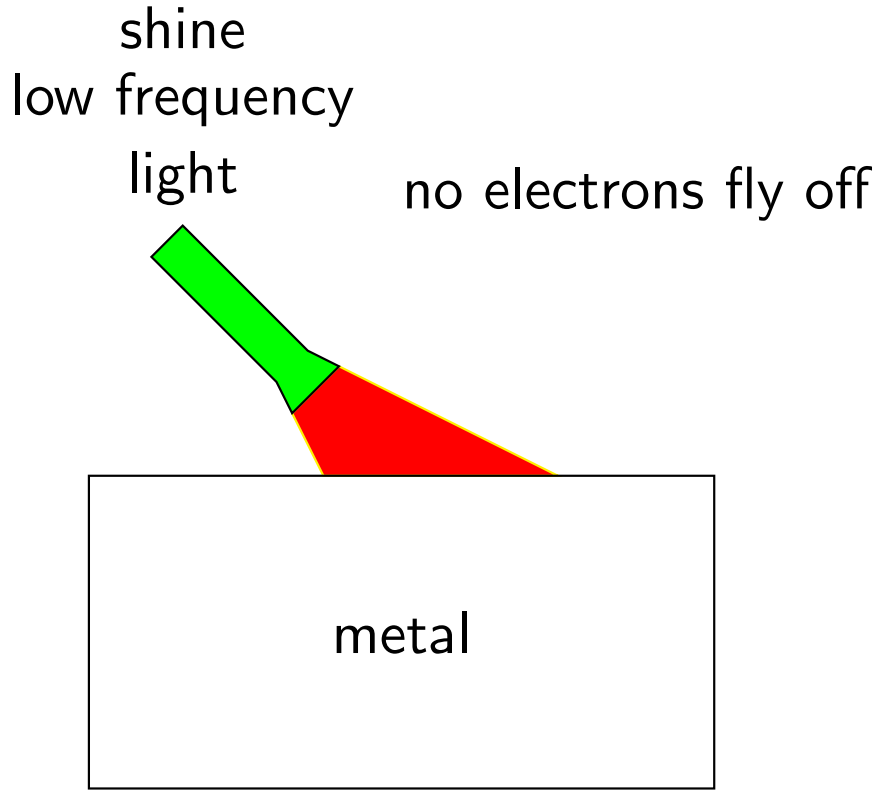


- Light provides the energy needed to free electrons from the metal.

The photoelectric effect



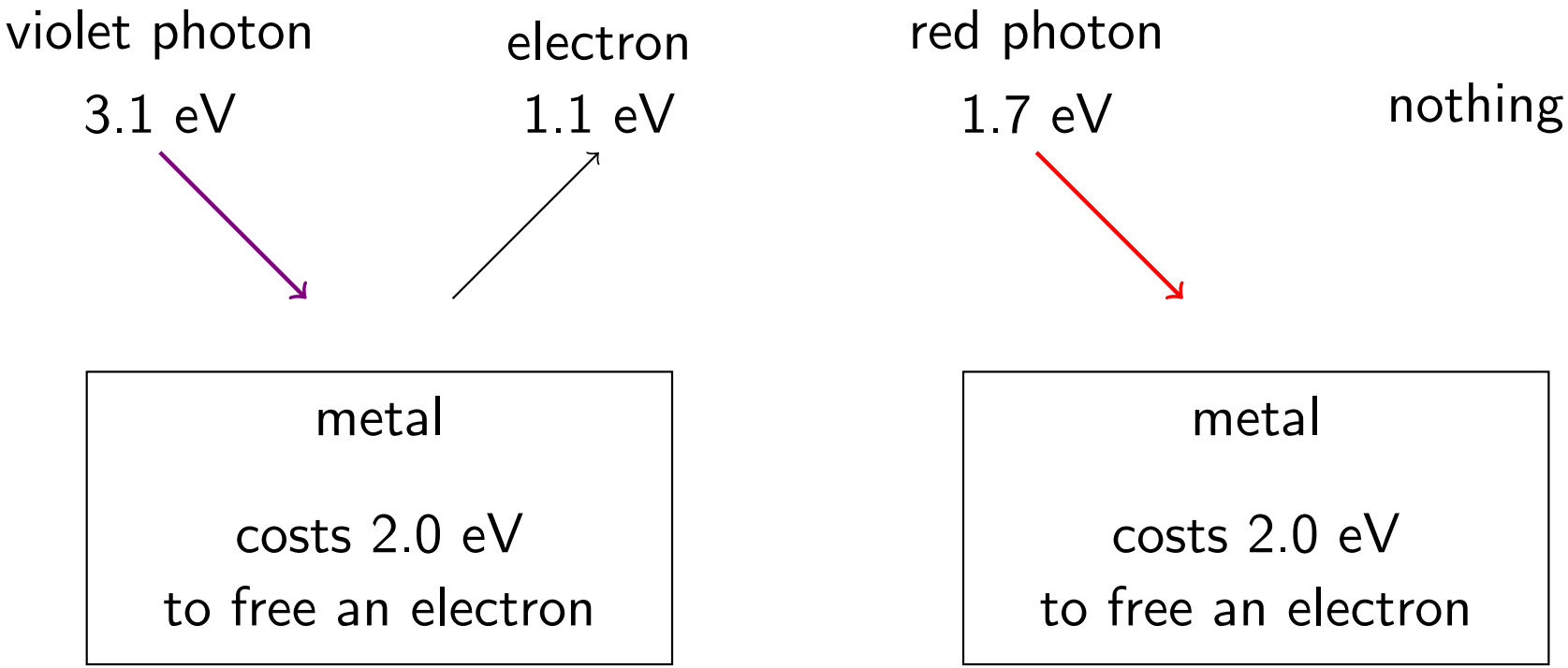
- violet has high frequency



- even if light is very bright

The photoelectric effect: Einstein's idea

- One photon must free one electron.
- Energy of one photon is $E = hf$.



How to convert among photon wavelength, frequency, and energy

Quantity	Symbol	Unit
Wavelength	λ	m, nm
Frequency	f	Hz, THz
Energy	E	J, eV

$$f = \frac{c}{\lambda}$$

$$E = hf$$

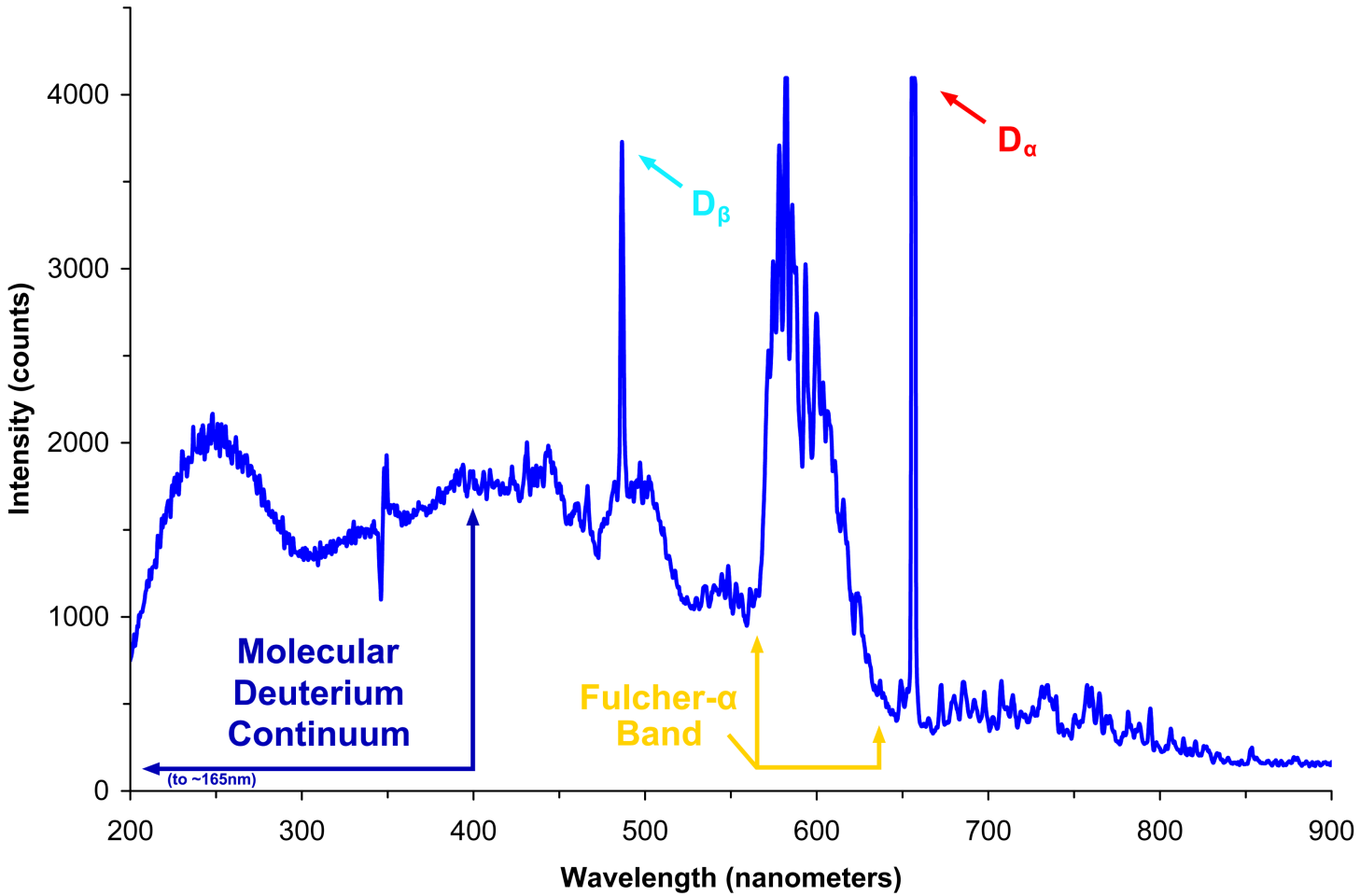
$$c = 3 \times 10^8 \ \mathrm{m/s}$$

$$h = 6.626 \times 10^{-34} \ \mathrm{J\ s} = 4.136 \times 10^{-15} \ \mathrm{eV\ s}$$

Exercise: Find the wavelength and frequency of a 1.7-eV photon

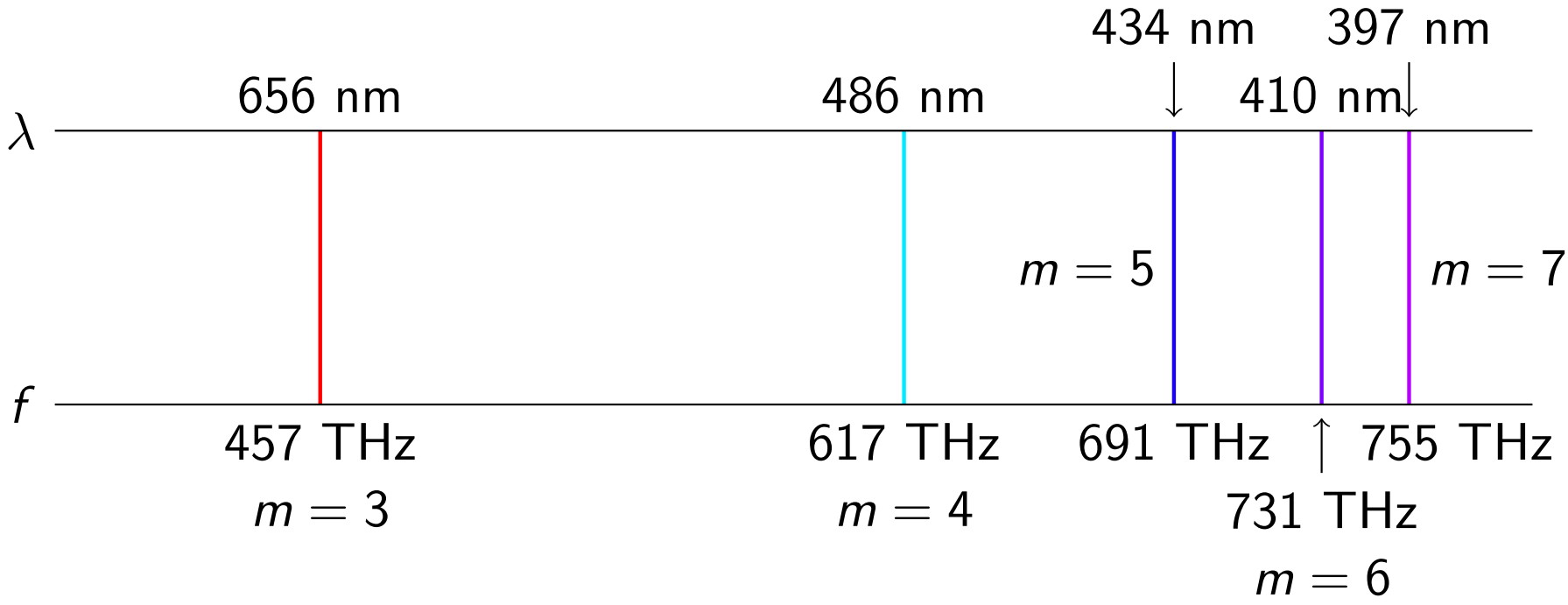
A red photon has an energy of 1.7 eV. What is its frequency? What is its wavelength?

Balmer: Hydrogen spectrum has sharp lines



- unlike a blackbody

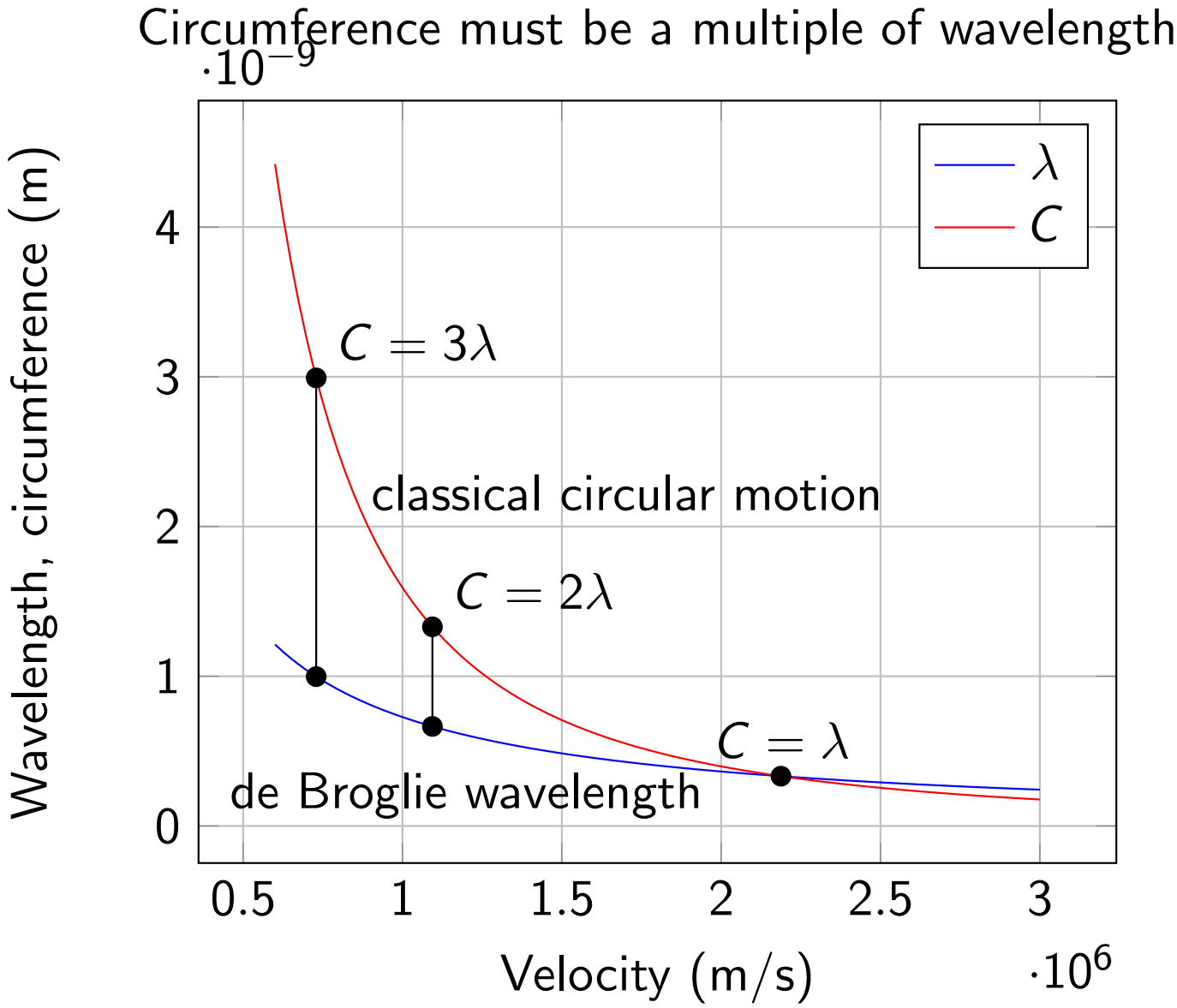
Balmer (1885) looked at 5 lines from hydrogen



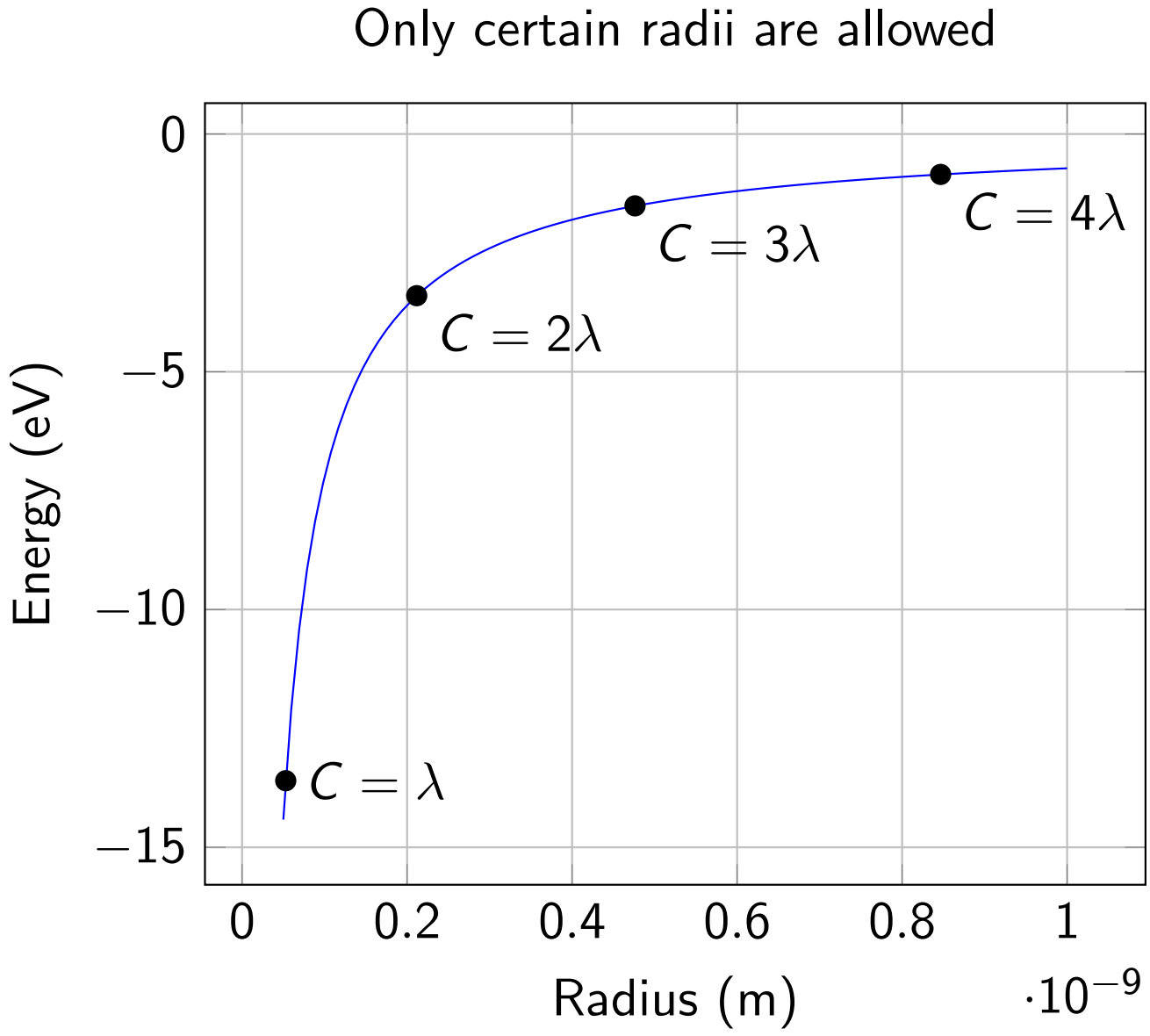
He found a pattern:

$$\lambda = \frac{365\ \mathrm{nm}}{1 - \frac{4}{m^2}} \qquad f = 3289\ \mathrm{THz}\left(\frac{1}{2^2} - \frac{1}{m^2}\right)$$

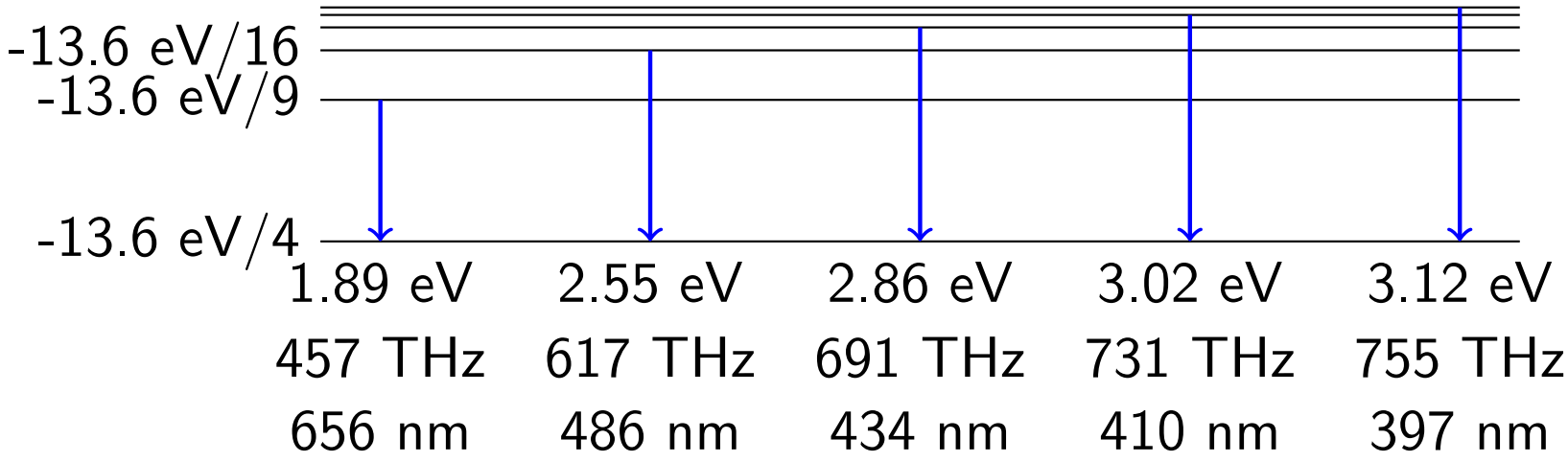
Bohr atom (1913)



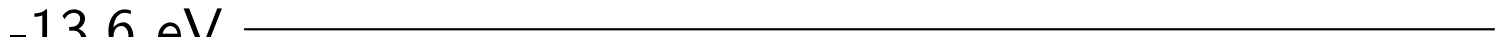
Bohr model produces hydrogen energy levels



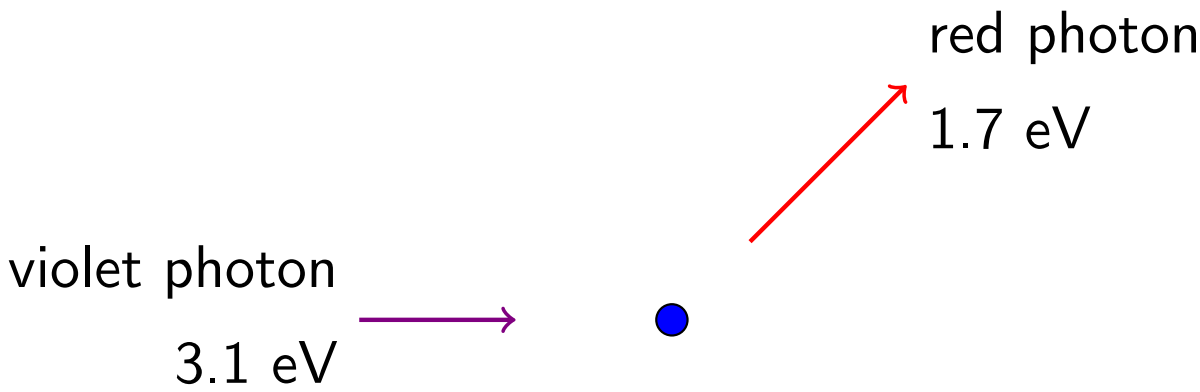
Hydrogen energy levels



Differences in energy levels
match the Balmer frequencies



Compton scattering



Four theories of light

- Light is a ray
  - Geometrical Optics
- Light is a wave (Young, 1803)
  - Wave Optics
- Light is an electromagnetic wave (Maxwell, 1865)
  - Classical Electromagnetic Theory (EM Theory)
- Light is a quantum field
  - Photon Theory (starting with Planck and Einstein, 1900–1905)
  - QED (Quantum Electrodynamics, 1949)

Matrices

Addition is commutative.

$$\begin{bmatrix} 2 & 6 \\ 8 & 7 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 9 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 9 & 12 \end{bmatrix}$$

Multiplication is not commutative.

$$\begin{bmatrix} 2 & 6 \\ 8 & 7 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 2 \cdot 4 + 6 \cdot 1 & 2 \cdot 3 + 6 \cdot 5 \\ 8 \cdot 4 + 7 \cdot 1 & 8 \cdot 3 + 7 \cdot 5 \end{bmatrix} = \begin{bmatrix} 14 & 36 \\ 39 & 59 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} 4 \cdot 2 + 3 \cdot 8 & 4 \cdot 6 + 3 \cdot 7 \\ 1 \cdot 2 + 5 \cdot 8 & 1 \cdot 6 + 5 \cdot 7 \end{bmatrix} = \begin{bmatrix} 32 & 45 \\ 42 & 41 \end{bmatrix}$$

Exercise: Multiply matrices

$$\begin{bmatrix} 6 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 8 & 3 \\ 5 & 7 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} =$$