Matrices

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Matrices

Addition is commutative.

$$\begin{bmatrix} 2 & 6 \\ 8 & 7 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 9 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 9 & 12 \end{bmatrix}$$

Multiplication is not commutative.

$$\begin{bmatrix} 2 & 6 \\ 8 & 7 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 2 \cdot 4 + 6 \cdot 1 & 2 \cdot 3 + 6 \cdot 5 \\ 8 \cdot 4 + 7 \cdot 1 & 8 \cdot 3 + 7 \cdot 5 \end{bmatrix} = \begin{bmatrix} 14 & 36 \\ 39 & 59 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} 4 \cdot 2 + 3 \cdot 8 & 4 \cdot 6 + 3 \cdot 7 \\ 1 \cdot 2 + 5 \cdot 8 & 1 \cdot 6 + 5 \cdot 7 \end{bmatrix} = \begin{bmatrix} 32 & 45 \\ 42 & 41 \end{bmatrix}$$

Exercise: Multiply matrices

$$\begin{bmatrix} 6 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 8 & 3 \\ 5 & 7 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} =$$

Pauli matrices

$$\Sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \Sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \Sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Two ways to write a 2×2 matrix

- Standard form
$$A = \begin{bmatrix} 7 & 2-i \\ 2+i & 3 \end{bmatrix}$$
- Pauli form
$$A = 5 \cdot 1 + 2\Sigma_1 + \Sigma_2 + 2\Sigma_3$$

To convert from Pauli form to standard form, substitute

$$
\begin{aligned}
A &= 5 \cdot 1 + 2\Sigma_1 + \Sigma_2 + 2\Sigma_3 \\
&= 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + 1 \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \\
&= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \\
&= \begin{bmatrix} 7 & 2-i \\ 2+i & 3 \end{bmatrix}
\end{aligned}
$$

To convert from standard form to Pauli form, use page 30

$$z_0 = \frac{1}{2}(a_{11} + a_{22}) \qquad z_1 = \frac{1}{2}(a_{12} + a_{21})$$

$$z_2 = \frac{i}{2}(a_{12} - a_{21}) \qquad z_3 = \frac{1}{2}(a_{11} - a_{22})$$