Complex Numbers

Scott N. Walck

October 5, 2018

Complex numbers are pairs of real numbers

Write ordered pair (3,4) as

$$3 + 4i.$$

- We call 3 the *real part* of the complex number.
- We call 4 the *imaginary part* of the complex number.

Addition of Complex Numbers

$$3 + 4i + 5 + 6i = 8 + 10i$$

Subtraction of Complex Numbers

$$3 + 4i - (5 + 6i) = -2 - 2i$$

Multiplication of Complex Numbers

Use FOIL with $i^2 = -1$.

$$(3 + 4i)(5 + 6i) = 15 + 18i + 20i + 24i^2 = -9 + 38i$$

Try these

$$z_1 = 2 + i$$

$$z_2 = 2 - 3i$$

Find

- $z_1 + z_2$
- $z_1 - z_2$
- $z_1 z_2$

The Complex Conjugate

If

$$z = 3 + 4i$$

we define

$$z^* = 3 - 4i.$$

z^* is called the *complex conjugate* of z.

- Replace i by $-i$.

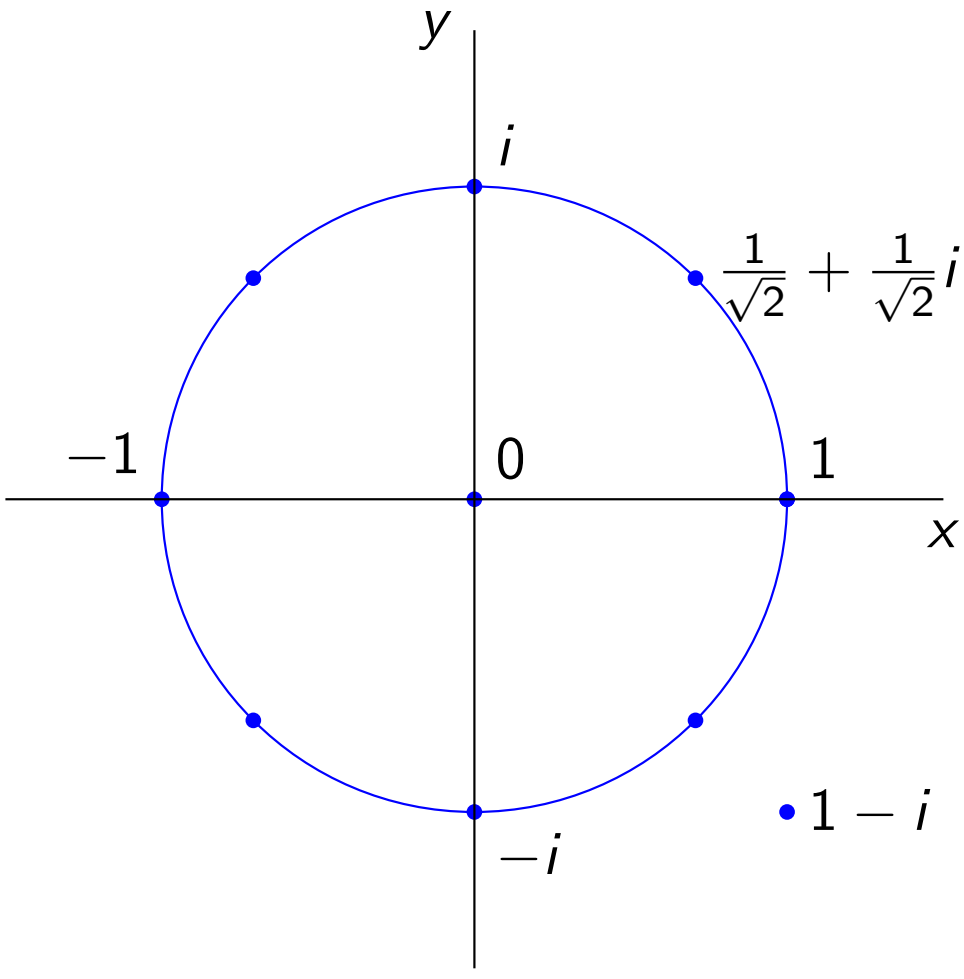
If you multiply any complex number by its complex conjugate, you get a real number.

$$(x + yi)(x - yi) = x^2 + y^2$$

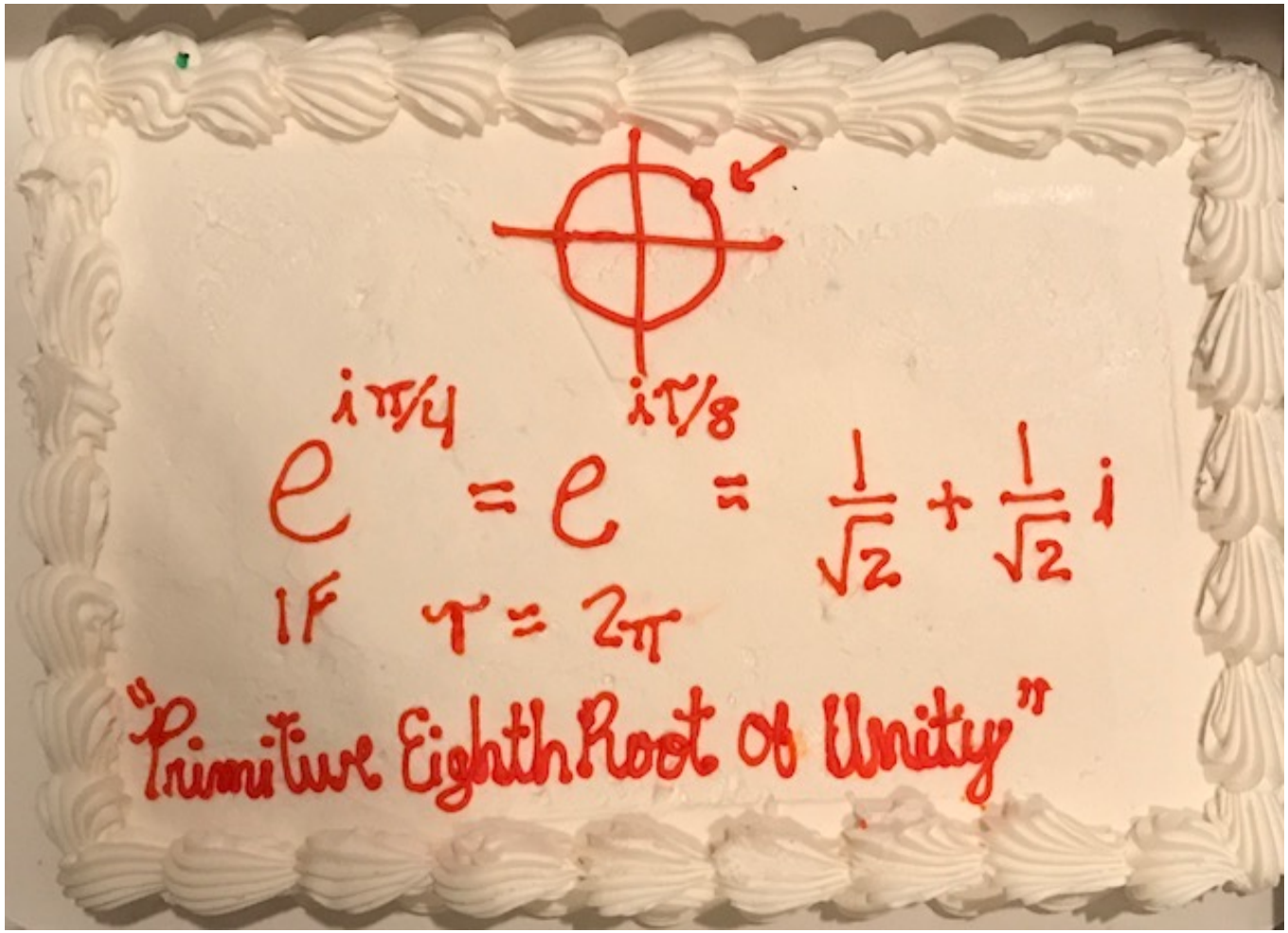
Division of Complex Numbers

$$\begin{aligned}
\frac{3+4i}{5+6i} &= \frac{3+4i}{5+6i} \cdot \frac{5-6i}{5-6i} \\
&= \frac{15-18i+20i-24i^2}{25-30i+30i-36i^2} \\
&= \frac{39+2i}{61} \\
&= \frac{39}{61} + \frac{2}{61}i
\end{aligned}$$

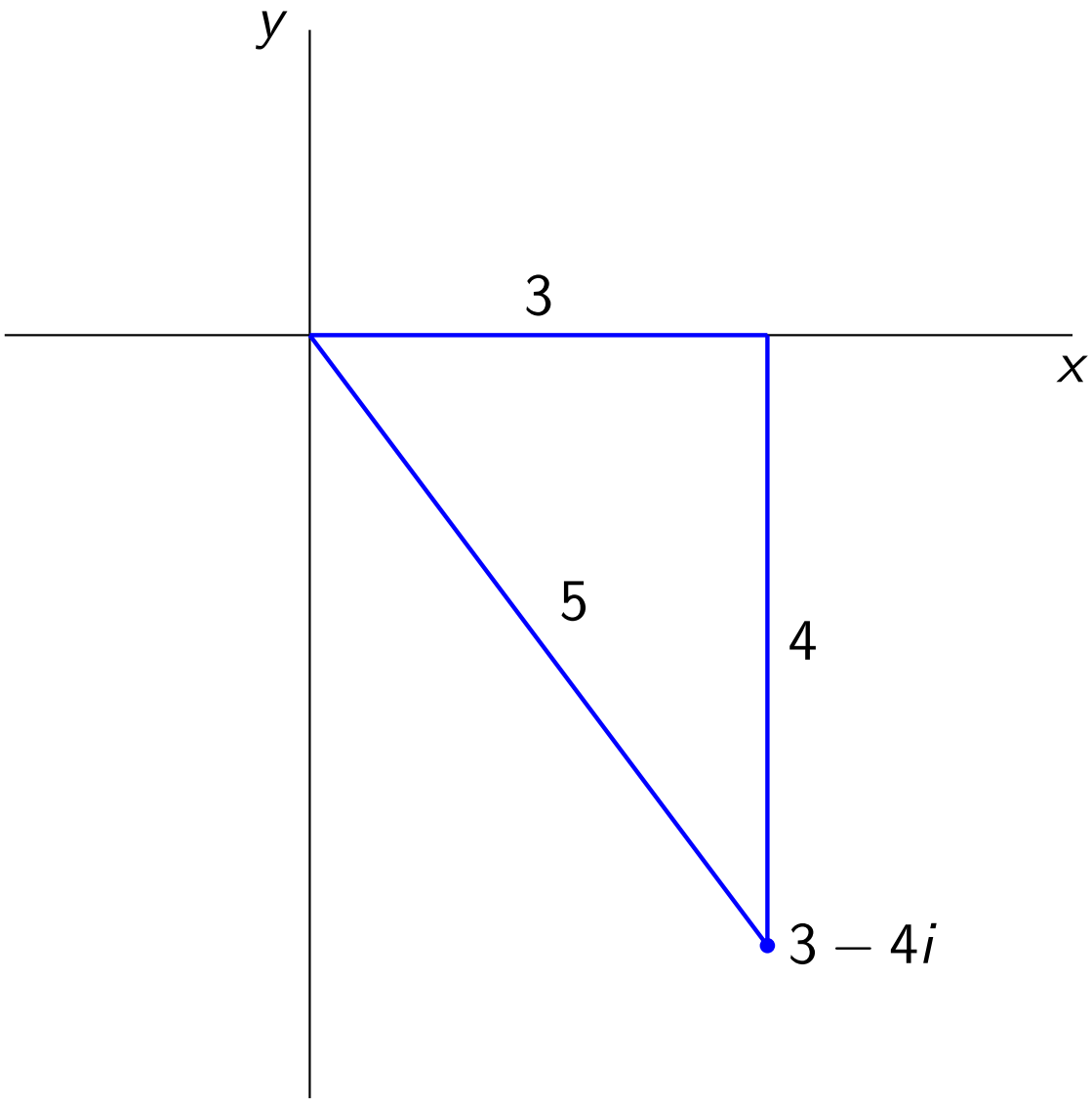
The Complex Plane



The cake for Scott's 51st birthday



Magnitude of a Complex Number



$$|3-4i| = \sqrt{3^2+(-4)^2}$$
$$= 5$$

More examples of magnitude

- Magnitude = absolute value

$$|1| = 1 \qquad |-1| = 1$$
$$|i| = 1 \qquad |-i| = 1$$
$$|4i| = 4 \qquad |-4i| = 4$$
$$|1+i| = \sqrt{2} \qquad |1-2i| = \sqrt{5}$$