Title: Pseudofree Finite Group Actions on 4-Manifolds.

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Abstract: We prove several theorems about the pseudofree, locally linear and homologically trivial action of finite groups G on closed, connected, oriented 4-manifolds M with non-zero Euler characteristic ($\chi(M) \neq 0$). In this setting, the $\text{rank}_p(G) \leq 1$, for $p \geq 5$ prime and $\text{rank}(G) \leq 2$, for $p = 2, 3$ (by [1]).

1. If a non-trivial finite group G acts on M in the above way, then $b_1(M) = 0$, and if $b_2(M) \geq 3$, then G must be cyclic and acts semi-freely.

2. If $b_2(M) = 2$ and $G = \mathbb{Z}_2 \times \mathbb{Z}_2$, then M must have the same 2-local homology and intersection form as $S^2 \times S^2$. If $G = \mathbb{Z}_q \rtimes \mathbb{Z}_2$, q odd, is non-abelian, then M must have the same q-local homology as $S^2 \times S^2$.

3. If $b_2(M) = 1$ and $G = \mathbb{Z}_3 \times \mathbb{Z}_3$, then M must have the same 3-local homology and intersection form as $\mathbb{CP}^2$. If $G = \mathbb{Z}_q \rtimes \mathbb{Z}_3$, q odd, $3 \nmid q$ is non-abelian, then M must have the same q-local homology as $\mathbb{CP}^2$.

4. If $b_2(M) = 0$ and $G = \mathbb{Z}_q \rtimes \mathbb{Z}_{2^r}$, q odd, $r \geq 2$ is non-abelian, then M must have the same q-local homology as S^4.

We combine these results into two main theorems: Theorem A and Theorem B in Chapter 1 of the thesis. These results strengthen the work done by Edmonds [2], and Hambleton and Pamuk [1]. We remark that for $b_2(M) \leq 2$ there are other examples of finite groups which can act in the above way (see [3] and section 2.4 of the thesis).

References

[1] Ian Hambleton and Semra Pamuk, *Rank conditions for finite group actions on 4-manifolds*, Canadian Journal of Mathematics. Journal Canadien de Mathématiques **74** (2022), no. 2, 550–572. MR 4411001

[2] Allan L. Edmonds, *Homologically trivial group actions on 4-manifolds*, arxiv:math/9809055 (1998).

[3] Michael P. McCooey, *Groups that act pseudofreely on $S^2 \times S^2$*, Pacific Journal of Mathematics **230** (2007), no. 2, 381–408. MR 2309166