Integral Surfaces, Prescribed Curvature, and Optics

Abstract. I will use the language of integral surfaces to discuss the general problem of trying to find a surface perpendicular to a vector field $\mathbf{W}$. I will examine the quantity $\text{curl}(\mathbf{W}) \cdot \mathbf{W}$ and use it to show existence, non-existence, or isolation of a perpendicular surfaces. In the case where there is no perpendicular surface, I will discuss the question of how to find the surface that is as "perpendicular as possible" to $\mathbf{W}$. I will use the Euler-Lagrange equations for the cost functional

$$\mathfrak{F}(\sigma) = \iint_{\sigma} |\mathbf{W} - \hat{n}|^2 \, dS$$

where σ is a surface with unit normals $\hat{n}$, to give a PDE for this surface. This PDE can be solved numerically, but is highly dependent on the boundary value conditions. Finally, I will show application of this method to the passenger-side vehicle mirror problem.