

MAS 170 Elementary Statistics Spring 2020
Exam 3 (Ch 13--18) Solutions

Questions 1 and 2.

$P(\text{Red shows 4 AND Blue shows 1})$
 $= P(\text{Red shows 4}) * P(\text{Blue shows 1})$ (multiplication rule for independent events)
 $= 1/6 * 1/6$

Question 3.

$P(\text{one die shows 4 and other shows 1})$
 $= P([R \text{ is 4 and B is 1}] \text{ OR } [R \text{ is 1 and B is 4}])$ (rephrasing)
 $= P(R \text{ is 4 and B is 1}) + P(R \text{ is 1 and B is 4})$ (addition rule)
 $= 1/6 * 1/6 + 1/6 * 1/6$ (same work as Questions 1 and 2)
 $= (\text{approx}) 5.56\%$

Questions 4 and 5.

Method 1.

$P(\text{total is 5})$
 $= P([one \text{ is 4 and other is 1}] \text{ OR } [one \text{ is 2 and other is 3}])$
(rephrasing)
 $= P(one \text{ is 4 and other is 1}) + P(one \text{ is 2 and other is 3})$
(addition rule)
 $= 2 * (\text{answer to question 3})$ (same reasoning as Question 3)
 $= 4/36$

Method 2.

Using the 6 by 6 chart of all 36 equally likely dice rolls (like in the textbook), we can see all four possible rolls that produce a total sum of 5 spots, namely, the rolls (4,1), (3,2), (2,3), and (1,4). So the $P(\text{total is 5}) = 4/36$.

Question 6.

We have $P(E) = 4/36 = 1/9$ (by Questions 4 and 5), but $P(E|F) = P(E \text{ is 1}) = 1/6$. Since $1/9$ does not equal $1/6$, we conclude that E,F are dependent.

Question 7.

Using the binomial formula with $n=18$, $k=3$, and $p=1/6$, the desired probability is

$(18 \text{ choose } 3) * (1/6)^3 * (5/6)^{15}$
 $= 816 * 5^{15} / 6^{18}$
 $= (\text{approx}) 24.52\%$

which rounds to 25% (to the nearest percent).

Question 8.

We expect to get a 4 on 1 out of 6 rolls of a fair die, which is

$1/6 = (\text{approx}) 16.67\%$ of the time.

To get 20% or more 4's, we desire a high relative error. The Law of Averages says that relative error goes down as the number of rolls goes up, so we have a better chance of winning with 200 rolls.

Questions 9 and 10.

The box model has 6 tickets: 1 ticket is a 1 and 5 tickets are 0s. We calculate the necessary numbers.

$\text{ave}(\text{box}) = 1/6 = (\text{approx}) .167$
 $\text{SD}(\text{box}) = \sqrt{5}/6 = (\text{approx}) .373$
 $\text{expected}(\text{sum}) = 1/6 * 200 = \text{approx } 33.33$
 $\text{SE}(\text{sum}) = \sqrt{5}/6 * \sqrt{200} = \text{approx } 5.27$
 $z = (40 - \text{expected}(\text{sum})) / \text{SE}(\text{sum}) = \text{approx } 1.27$
 $P = (\text{approx}) (100 - 80)/2 = 10\%$

Questions 11 and 12.

The box model has 6 tickets: 1 ticket is a +\$3, and 5 tickets are -\$1. Here are the calculations.

```
ave(box) = -1/3 = (approx) -.33
SD(box) = 4 * sqrt(1/6 * 5/6) = approx 1.49
expected(sum) = -1/3 * 200 = (approx) -$66.67
SE(sum) = 2/3 * sqrt(5) * sqrt(200) = (approx) $21.08
z = (approx) (-20 - (-66.67)) / 21.08 = (approx) 2.21
P = (approx) (50 + 1/2 * 97.3) = 98.7%
```

The last number rounds to 99% (to the nearest percent).

Questions 13 and 14.

The box model has 6 tickets, labeled 1,2,3,4,5,6.

```
ave(box) = 3.5
SD(box) = sqrt((2.5^2 + 1.5^2 + .5^2)/3) = (approx) 1.71
expected(sum) = 3.5 * 200 = 700
SE(sum) = (approx) 1.71*sqrt(200) = (approx) 24.2
z = (approx) 20/24.2 = (approx) .83
P = (approx) 59%
```